

Shannon Capacity of Wireless Channels

Lecture Outline

- **Shannon Capacity**
- **Capacity of Flat-Fading Channels**
- **Capacity with Fading Known at Receiver**
- **Capacity with Fading Known at Transmitter and Receiver**
- **Optimal Rate and Power Adaptation (Water Filling)**
- **Channel Inversion with Fixed Rate Transmission**

1. Shannon Capacity

- The maximum mutual information of a channel. Its significance comes from Shannon's coding theorem and converse, which show that capacity is the maximum error-free data rate a channel can support.
- Capacity is a channel characteristic - not dependent on transmission or reception techniques or limitation.
- In AWGN, $C = B \log_2(1 + \gamma)$ bps, where B is the signal bandwidth and $\gamma = P/(N_0B)$ is the received signal-to-noise power ratio.

2. Capacity of Flat-Fading Channels:

- Depends on what is known about the channel.
- Three cases: 1) Fading statistics known; 2) Fade value known at receiver; 3) Fade value known at transmitter and receiver.
- When only fading statistics known, capacity difficult to compute. Only known results are for Finite State Markov channels, Rayleigh fading channels, and block fading.

3. Fading Known at the Receiver:

- Capacity given by $C = \int_0^\infty B \log_2(1 + \gamma)p(\gamma)d\gamma$ bps, where $p(\gamma)$ is the distribution of the fading SNR γ .
- By Jensen's inequality this capacity always less than that of an AWGN channel.
- "Average" capacity formula, but transmission rate is fixed.

4. Capacity with Fading Known at Transmitter and Receiver

- For fixed transmit power, same capacity as when only receiver knows fading.
- Transmit power as well as rate can be adapted.
- Under variable rate and power $C = \max_{P(\gamma): \int P(\gamma)p(\gamma)d\gamma = \bar{P}} \int_0^\infty B \log_2 \left(1 + \frac{P(\gamma)\gamma}{\bar{P}}\right) p(\gamma)d\gamma$, where $P(\gamma)$ is power adaptation

5. Optimal Power and Rate Adaptation

- Optimal adaptation found via Lagrangian differentiation.
- Optimal power adaptation is a “water-filling” in time: power $P(\gamma) = \gamma_0^{-1} - \gamma^{-1}$ increases with channel quality γ above an optimized cutoff value γ_0 .
- Rate adaptation relative to $\gamma \geq \gamma_0$ is $B \log_2(\gamma/\gamma_0)$: also increases with γ above cutoff.
- Resulting capacity is $C = \int_{\gamma_0}^{\infty} B \log_2(\gamma/\gamma_0) p(\gamma) d\gamma$.
- Capacity with power and rate adaptation not much larger than when just receiver knows channel, but has lower complexity and yields more insight into practical schemes.
- Capacity in flat-fading can exceed the capacity in AWGN, typically at low SNRs.

6. Channel Inversion

- Suboptimal transmission strategy where fading is inverted to maintain constant received SNR.
- Simplifies system design and is used in CDMA systems for power control.
- Capacity with channel inversion greatly reduced over that with optimal adaptation (capacity equals zero in Rayleigh fading).
- Truncated inversion: performance greatly improved by inverting above a cutoff γ_0 .

Main Points

- Capacity of flat-fading channels depends on what is known about the fading at receiver and transmitter.
- Capacity when only the receiver knows the fading is an average of capacity in AWGN, averaged over the fading distribution.
- Capacity when both transmitter and receiver known channel requires optimal adaptation relative to each channel state. Capacity increases when transmitter also knows the channel only when power is adapted.
- Capacity-achieving transmission scheme uses variable-rate variable-power transmission with power water-filling in time.
- Power and rate adaptation does not significantly increase capacity, and rate adaptation alone yields no increase. These results may not carry over to practical schemes.
- Channel inversion practical but has poor performance. Performance can be significantly improved by truncating.