Linear Digital Modulation and its Performance in AWGN and in Fading

Lecture Outline

- **Review of Linear Modulation**
- **Performance of Linear Modulation in AWGN**
- **Performance Metrics in Flat Fading**
- **Outage Probability**
- **Average Probability of Error**

1. **Linear Modulation**
   - Over the ith symbol period, bits are encoded in carrier amplitude or phase $s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$, where $\phi_1(t) = g(t)\cos(2\pi f_c t + \phi_0)$ and $\phi_2(t) = g(t)\sin(2\pi f_c t + \phi_0)$ for initial phase offset ϕ_0.
   - Pulse shape $g(t)$ determines signal bandwidth, and is typically Nyquist.
   - Baseband representation is $s(t) = \Re\{x(t)e^{j\phi_0}e^{j2\pi f_c t}\}$ for $x(t) = (s_{i1} + js_{i2})g(t)$.
   - The constellation point (s_{i1}, s_{i2}) has M possible values, hence there are $\log_2 M$ bits per symbol.

2. **Performance of Linear Modulation in AWGN:**
   - ML detection corresponds to decision regions.
   - For coherent modulation, probability of symbol error P_s depends on the number of nearest neighbors α_M, and the ratio of their distance d_{min} to the square root $\sqrt{N_0}$ of the noise power spectral density (this ratio is a function of the SNR γ_s).
   - P_s approximated by $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$, where β_M depends on the modulation.
   - Alternate Q function representation $Q(z) = \frac{1}{\pi}\int_0^{.5\pi} \exp[-z^2/(2\sin^2\phi]d\phi$ leads to closed form expression for error probability of PSK in AWGN and, more importantly, greatly simplifies fading/diversity analysis.

3. **Performance of Linear Modulation in Fading:**
   - In fading γ_s and therefore P_s are random variables.
   - Three performance metrics to characterize the random P_s.
   - Outage: $p(P_s > P_{target} = p(\gamma < \gamma_{target})$
   - Average P_s $(\overline{P}_s = \int P_s(\gamma)p(\gamma)d\gamma)$.
   - Combined outage and average P_s.

4. **Outage Probability:** $p(P_s > P_{s_{\text{target}}}) = p(\gamma_s < \gamma_{s_{\text{target}}})$.
   - Outage probability used when fade duration long compared to a symbol time.
   - Obtained directly from fading distribution and target γ_s.
   - Can obtain simple formulas for outage in log-normal shadowing or in Rayleigh fading.

5. **Average P_s:** $\overline{P}_s = \int P_s(\gamma_s) p(\gamma_s) d\gamma_s$.
   - Rarely leads to close form expressions for general $p(\gamma_s)$ distributions.
   - Can be hard to evaluate numerically.
   - Can obtain closed form expressions for general linear modulation in Rayleigh fading (using approximation $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$ in AWGN).

Main Points

- Can approximate symbol error probability P_s of MPSK and MQAM in AWGN using simple formula: $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$, with standard or alternate Q function representation.
- In fading, P_s is a random variable, characterized by average value, outage, or combined outage and average.
- Outage probability based on target SNR in AWGN.
- Closed-form expressions for average P_s of BPSK and DPSK in Rayleigh fading decrease as $1/\overline{\gamma_b}$.
- Average P_s obtained by integrating P_s in AWGN over fading distribution. In general hard to compute for standard Q function since double-integral diverges analytically and numerically.
- Fading leads to large outage probability and greatly increased average P_s.