Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks

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Table of Contents

1 Introduction
   • What are we trying to solve?
   • What do we mean by “scaling laws”?  
   • Dense vs Extended Networks
   • Why is this problem important?

2 Previous Work
   • Previous Work: Dense Networks
   • Previous Work: Extended Networks

3 Achievable Scheme
   • Dense Networks
   • Extended Networks

4 Conclusions
   • Conclusions
   • Questions
What are we trying to solve?

- Consider $n$ source-destination pairs located randomly.
- Signals transmitted from one user to another at distance $r$ are subject to:
  - power loss $r^{-\alpha}$, where $\alpha \in [2, 6]$,
  - a random phase

How does information capacity scale as $n$ grows?
What do we mean by “scaling laws”? 

- Assume that each node wants to communicate to a random node at a rate $R(n)$ bits/sec.

Definition (Total throughput)

$$T(n) = nR(n)$$

- What is: $\max_{all \ schemes} T(n)$ as $n$ grows?
Dense vs Extended Networks

Definition (Dense networks)

Area is fixed and the density of nodes increases.

- Interference limited.
- Example: Cellular networks in urban areas.

Definition (Extended networks)

Density is fixed and the area increases.

- Coverage limited.
- Example: Cellular networks in rural areas.
- Power limitation come to play.
Why is this problem important?

- Theoretical curiosity
  - FlashForward:
    - In a dense network capacity scales linearly with $n$. !!

- Broad design directions for the engineers
  - FlashForward:
    - distributed MIMO communication
    - Node Cooperation
    - Hierarchical and Digital Architecture
    - Many long-range communications.
Table of Contents

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- Critical Assumption:
  Signals received from other nodes (except one) are regarded as noise.
  - nearest-neighbor multihop scheme $\rightarrow$ many retransmissions $\rightarrow$ scaling no better than $O(\sqrt{n})$. :-(

- Franceschetti et al. [4] proved that this bound is achievable.

Thus, Scaling law: $\Theta(\sqrt{n})$

Is this scaling law a consequence of the physical-layer technology or can we do better?

Yes we can!
Let me tell you how in a few slides!

- If $\alpha > 6$ then nearest neighbor multihop scheme is optimal.
- Many subsequent works that relaxed the condition down to $\alpha > 4$.
- What about $\alpha \in [2, 4]$? Is nearest neighbor multihop scheme is optimal?
  
  No!

Intuition: For $\alpha < 4$, the network is interference limited $\rightarrow$ like a dense network...
Table of Contents

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   • Conclusions
   • Questions
Problem Formulation

- $n$ nodes uniformly and i.i.d. in a square of unit area.
- Communication over flat channels
- No multipath effects and Line of sight type environment
- The channel gains are known to all the nodes.
- Far-Field Assumptions
- Path loss and random phase.
Problem Formulation

Upper bound

\[ T(n) = O(n \log(n)) \]

Main idea of the proof:
- The rate \( R(n) \) from any source node \( s \) is bounded by the capacity of the SIMO channel.

Achievable Rate

\[ T(n) \geq K_\epsilon n^{1-\epsilon}, \quad \forall \epsilon > 0. \]

Main idea of the proof:
- Construct clusters and perform long-range MIMO transmissions between clusters.
Divide the network in clusters of size $M$. Take at random a pair $(s, d)$. Assume nodes $s, d$ belong to clusters $S$ and $D$ respectively. Assume node $s$ needs to transmit $M$ bits to node $d$.

- **Phase 1: Setting up Transmit Cooperation**
  - Node $s$ distributes *locally* the $M$ bits to the nodes of the current cluster

- **Phase 2: MIMO Transmissions**
  - The nodes of the cluster $S$ cooperate and perform *long-range* transmission to all the nodes of the cluster $D$.

- **Phase 3: Cooperate to Decode**
  - Nodes in $D$ cooperate to decode the message and send it *locally* to $d$. 
Phase 1: Setting up Transmit Cooperation

- Clusters work in parallel.
- Inside each cluster, each node needs to distribute $M$ bits to the rest $M - 1$ nodes of the cluster. $\rightarrow M^2$ bits.
- Assume we have a transmission scheme that achieves $M^b$ bits-slot, where $0 \leq b < 1$.
- Therefore, we need $\frac{M^2}{M^b} = M^{2-b}$ time slots for phase 1.
Phase 2: MIMO Transmissions

- There are $n$ (s,d) pairs in all the network.
- The long-distance MIMO transmissions between the clusters are performed **one at a time**.
- We need $n$ time slots for phase 2.
Phase 3: Cooperate to Decode

- Clusters work in parallel.
- $M$ destination nodes in each cluster. →
  - Each cluster received $M$ transmissions in phase 2. →
  - Each node in the cluster received $M$ observations.
- Each node quantize each observation into $Q$ bits. → $QM^2$ bits need to be locally flooded inside the cluster.
- Therefore, we need $\frac{QM^2}{M^b} = QM^{2-b}$ time slots for phase 3.
Aggregate Throughput

\[ T(n) = \frac{nM}{M^{2-b} + n + QM^{2-b}} = \frac{1}{2+Q} n^{\frac{1}{2-b}} \]

- Note that \( \frac{1}{2-b} > b \), \( \forall \ 0 \leq b < 1 \).
- We started from a scheme with \( T(n) = n^b \).
- We have a new scheme that achieves \( T(n) = n^{\frac{1}{2-b}} > n^b \).
- By repeating this procedure we get:
  \[ T(n) = K_\epsilon n^{1-\epsilon} \]
Graphical Representation
Main Result

The same scheme achieves $T(n) \geq K \cdot n^{2-\frac{\alpha}{2-\epsilon}}$ for $2 \leq \alpha < 3$
(better than just multihop.)

“Bursty” modification of the hierarchical scheme:

- Density is fixed, area is $\sqrt{n} \times \sqrt{n}$ square. $\rightarrow$
- All distances increase by $\sqrt{n} \rightarrow$
- Received powers are all decreased by $n^{\frac{\alpha}{2}}$.
- Power constraint is $\frac{P}{n^{\frac{\alpha}{2}}}$
- Run the scheme a fraction $\frac{1}{n^{\alpha/2-1}}$ with power $\frac{P}{n}$.
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We achieved an optimal throughput performance for a dense network!
We used this scheme for the extended networks to fill in the gap for $\alpha \in [2, 4]$.

Main points:
- Node cooperation
- MIMO transmissions
- Hierarchical Cooperation
- Many long-range communications.
Questions ...
References


