

Application of Proximal Algorithms to Three Dimensional Deconvolution Microscopy

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Motivation

In microscopy, shot noise dominates image formation. This signal dependent noise follows a Poisson distribution, and the image formation is modeled as:

$$\mathbf{b} \sim \mathcal{P}(\mathbf{A}\mathbf{x})$$

The popular Richardson-Lucy algorithm solves the above problem via iterative corrections to the true image estimate. However, for large problems, it tends to converge slowly. Moreover, it is difficult to introduce non-differentiable priors into the algorithm.

Proximal Methods

Proximal methods are a class of iterative algorithms to solve large scale, multi-dimensional problems. Each iteration evaluates a proximal operator, which is a simple convex problem that can be solved quickly.

The following is the representation of the optimization problem that includes the maximum likelihood estimator for the Poisson noise term and the non-negativity constraint.

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && -\log(p(\mathbf{b}|\mathbf{z}_1)) + \mathcal{I}_{\mathbb{R}_+}(\mathbf{z}_2) \\ & \text{subject to} && \underbrace{\begin{bmatrix} \mathbf{A} \\ \mathbf{I} \end{bmatrix}}_{\mathbf{K}} \mathbf{x} - \underbrace{\begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}}_{\mathbf{z}} = 0 \end{aligned}$$

Chambolle-Pock Method

Chambolle-Pock is a primal-dual algorithm well suited for solving image deconvolution and denoising problems. The iterative updates for this specific case are as follows:

Initialization: $\bar{\mathbf{x}}^0 = \mathbf{x}^0 = \mathbf{b}, \sigma\tau < \|\mathbf{K}\|_2^2$

for $k = 1$ to MaxIter

$$\mathbf{z}^{k+1/2} \leftarrow \mathbf{z}^k + \sigma \mathbf{K} \bar{\mathbf{x}}^k$$

$$\mathbf{z}_1^{k+1} \leftarrow \mathbf{z}_1^{k+1/2} - \sigma \text{prox}_{\mathcal{P},\sigma}(\mathbf{z}_1^{k+1/2}/\sigma)$$

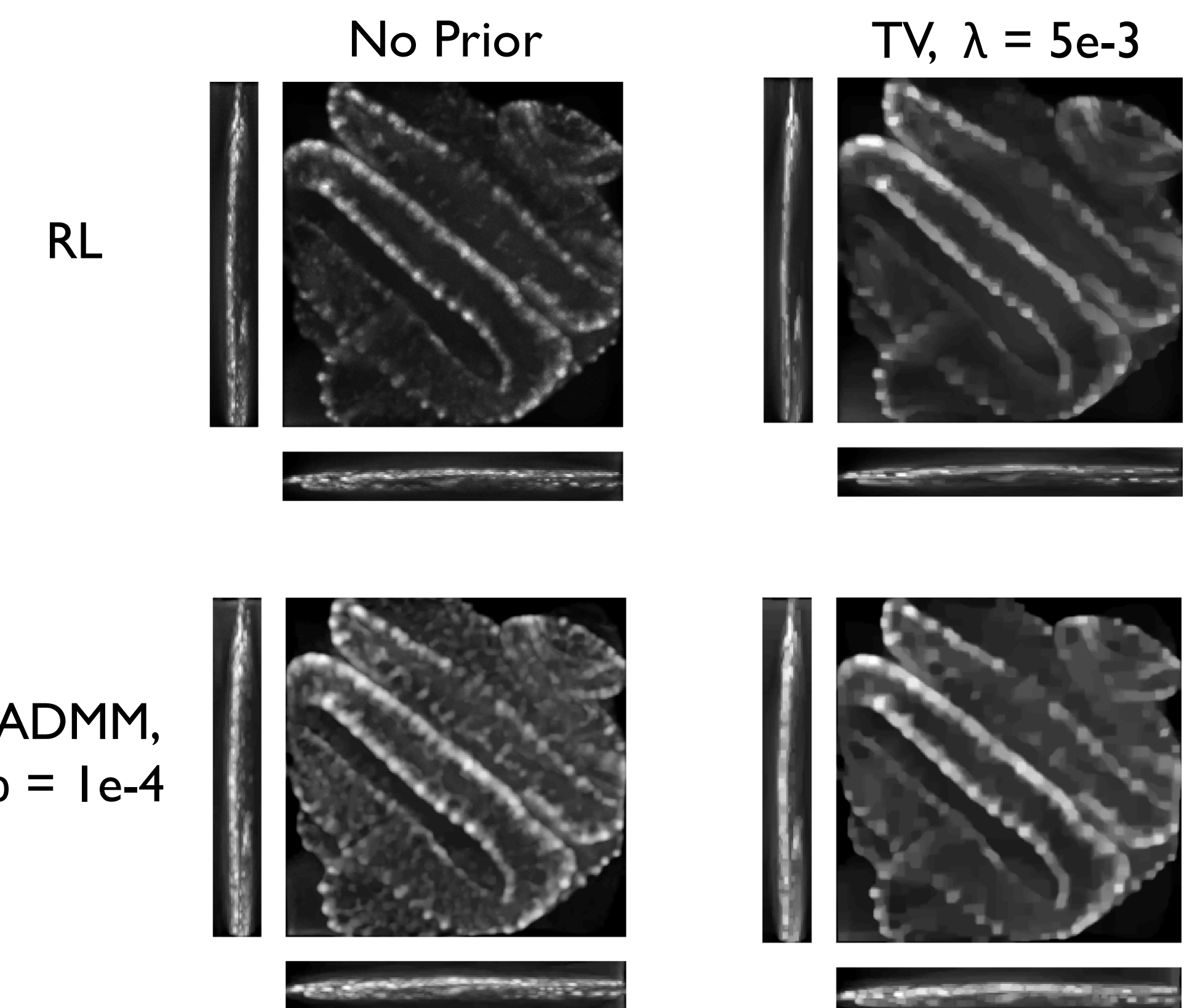
$$\mathbf{z}_2^{k+1} \leftarrow \mathbf{z}_2^{k+1/2} - \sigma \text{prox}_{\mathcal{I},\sigma}(\mathbf{z}_2^{k+1/2}/\sigma)$$

$$\mathbf{x}^{k+1} \leftarrow \mathbf{x}^k - \tau \mathbf{K}^T \mathbf{z}^{k+1}$$

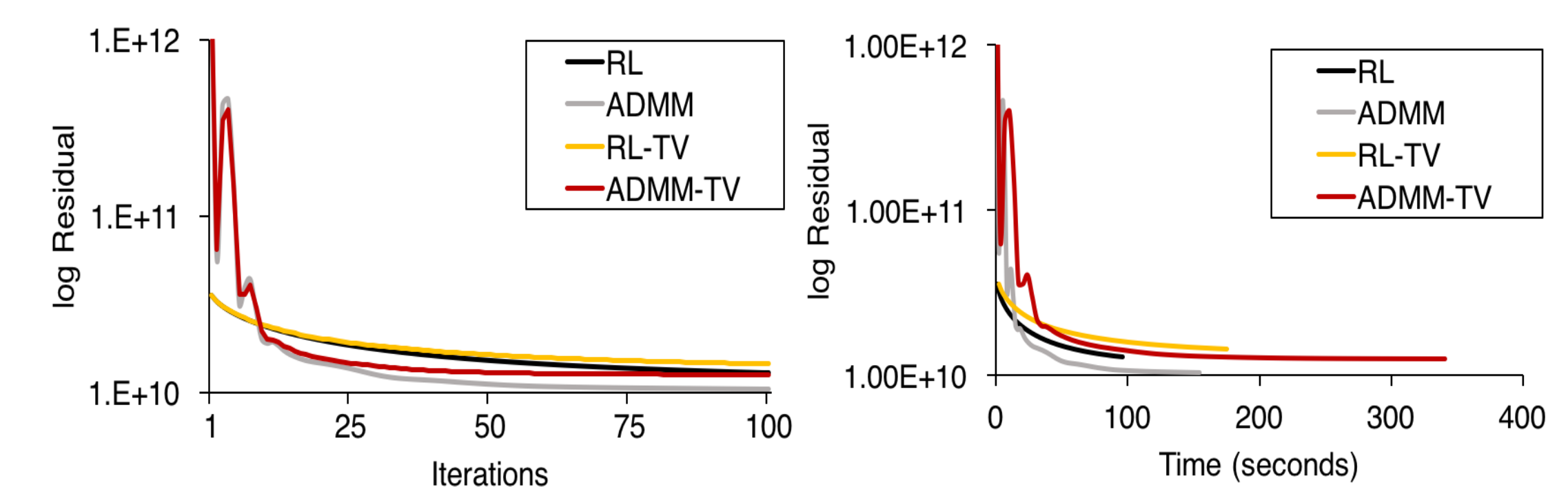
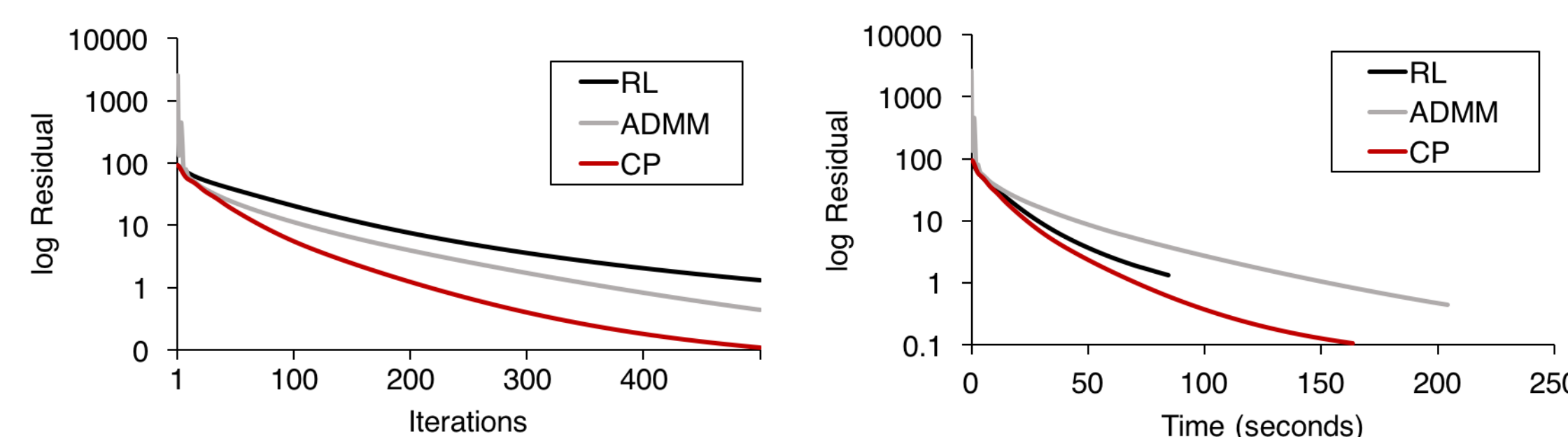
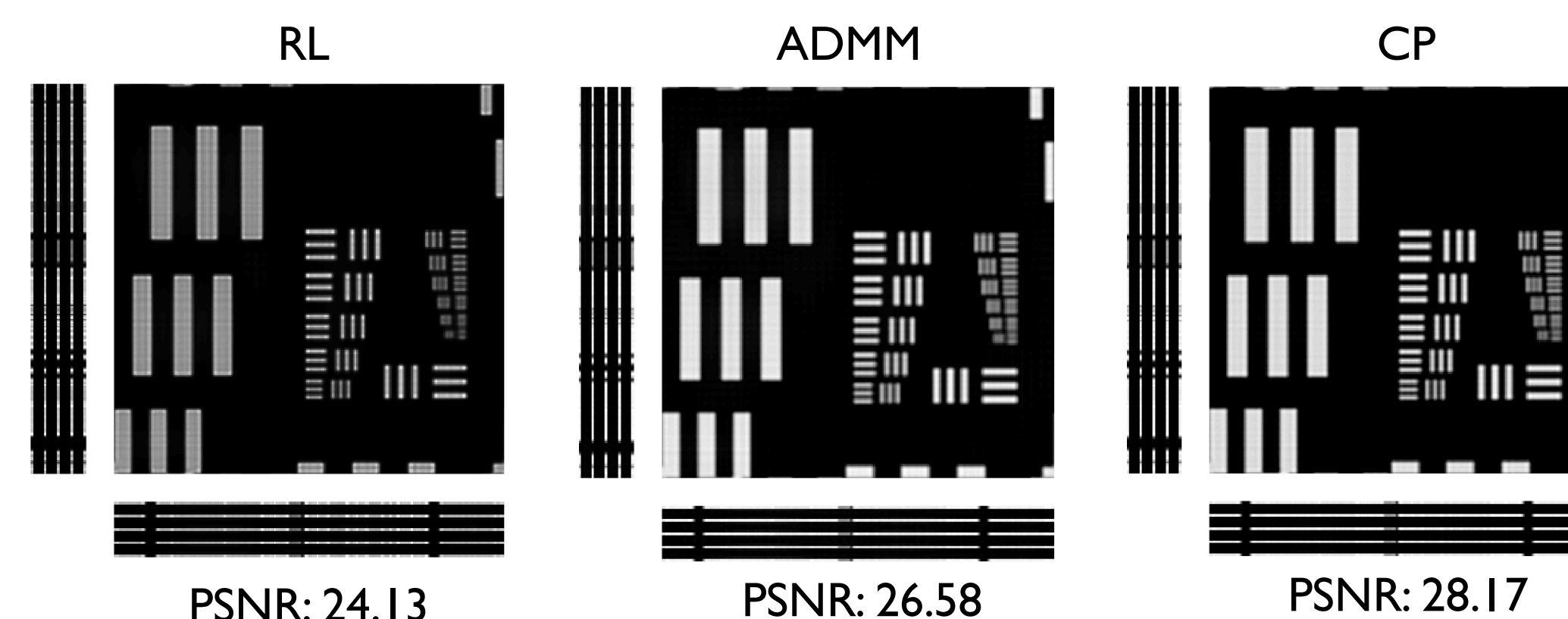
$$\bar{\mathbf{x}}^{k+1} \leftarrow 2\mathbf{x}^{k+1} - \mathbf{x}^k$$

end

Captured Results



Simulated Results



Related Work

Chambolle, A., & Pock, T. (2011). A first-order primal-dual algorithm for convex problems with applications to imaging. *Journal of Mathematical Imaging and Vision*.

Dupé, F. X., Fadili, M. J., & Starck, J. L. (2011, September). Inverse problems with Poisson noise: Primal and primal-dual splitting. In *Image Processing (ICIP)*, 2011.

Parikh, N., & Boyd, S. P. (2014). *Proximal Algorithms*. Foundations and Trends in optimization.