



Inverting for Temperature Distribution Inside a Glacier

Stanford
Radio Glaciology



Contact:
bienert@stanford.edu

Nicole Bienert

Stanford University Department of Electrical Engineering

Motivation

Approximately 80% of Antarctica's contribution to sea level rise comes from outlet glaciers and ice streams.

The physics controlling ice streams is not well understood when topographic forcing is absent.

Temperature structure is hypothesized to control the width of ice streams.[1]

No radar exists to measure temperature structure and test hypotheses related to temp.

Temperature Tomography w/ Radar

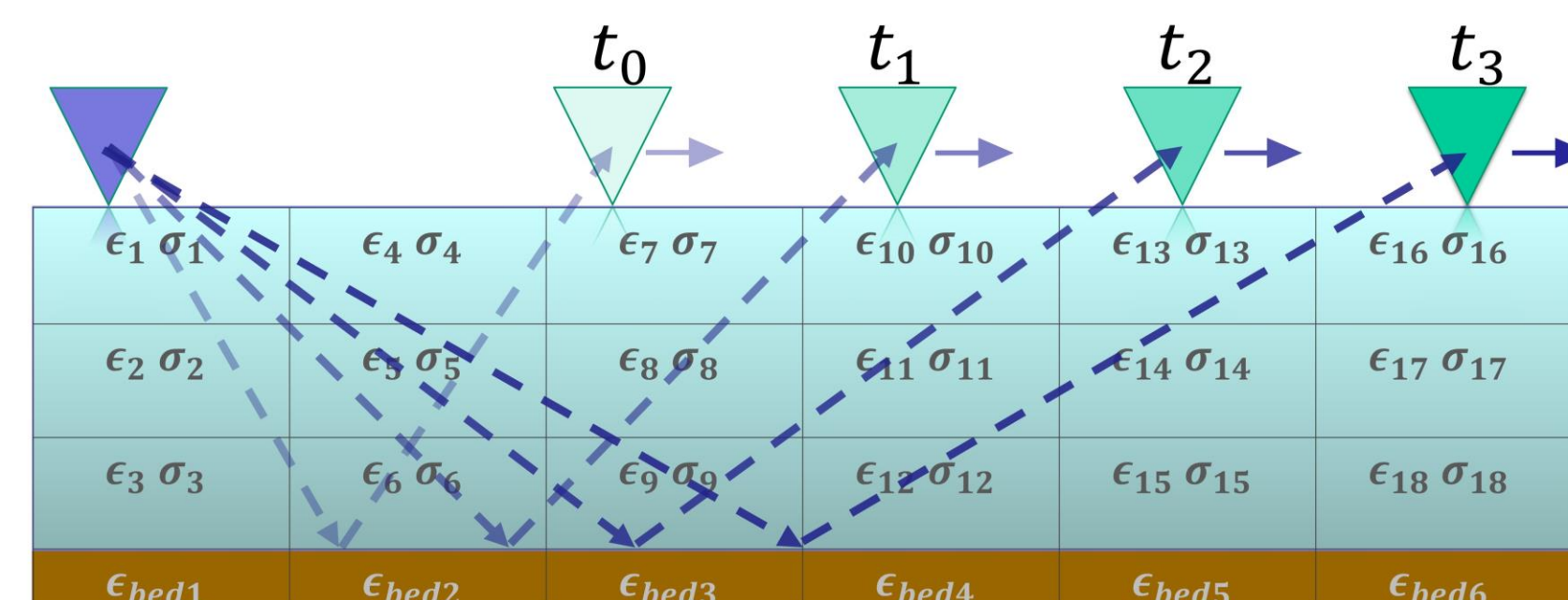
Goal: To demonstrate that bistatic measurements could be used with an inverse method such as ADMM to estimate the thermal structure to validate or reject hypotheses about ice physics.

Theory of Operation

Transmitter

Moving Receiver

Power received in a bistatic experiment can be used to invert for temperature.



$$\sigma = \sum_{i=0}^2 \sigma_0^i C_i e^{-\frac{E_i}{k} \left(\frac{1}{T} - \frac{1}{T_r} \right)} [2]$$

$$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon_1}} \quad (\text{low-loss})$$

$$P_{RX} = P_{TX} G_1 G_2 \rho \left(\frac{\lambda}{4\pi r} \right)^2 \Gamma^2 e^{-2\alpha r}$$

New Technique: Bistatic Radar w/ ADMM Inversion

Measure power reflected from bottom of glacier



Scale power to isolate conductivity

ADMM to invert for conductivity

Gradient descent to find temp from conductivity

$$\sum_{n=1}^{\# \text{ grids}} \sigma_n \Delta r_{k,n}$$

$$= \frac{-2 \ln(P_k)}{\sqrt{\frac{\mu}{\epsilon}} \ln \left(P_{TX} G^2(\theta_k) \rho \Gamma_k^2(\theta_k) \left(\frac{\lambda}{4\pi r_k} \right)^2 \right)}$$

$$\text{Min} \left\| \begin{bmatrix} \Delta r_{1,1} & \dots & \Delta r_{1,n} \\ \vdots & \ddots & \vdots \\ \Delta r_{k,1} & \dots & \Delta r_{k,n} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \right\|_2$$

$$- \lambda \left\| \begin{bmatrix} \nabla_x [\sigma_1 & \sigma_3] \\ \nabla_y [\sigma_2 & \dots \sigma_n] \end{bmatrix} \right\|_1$$

$$\sigma = \sum_{i=0}^2 \sigma_0^i C_i e^{-\frac{E_i}{k} \left(\frac{1}{T} - \frac{1}{T_r} \right)} [2]$$

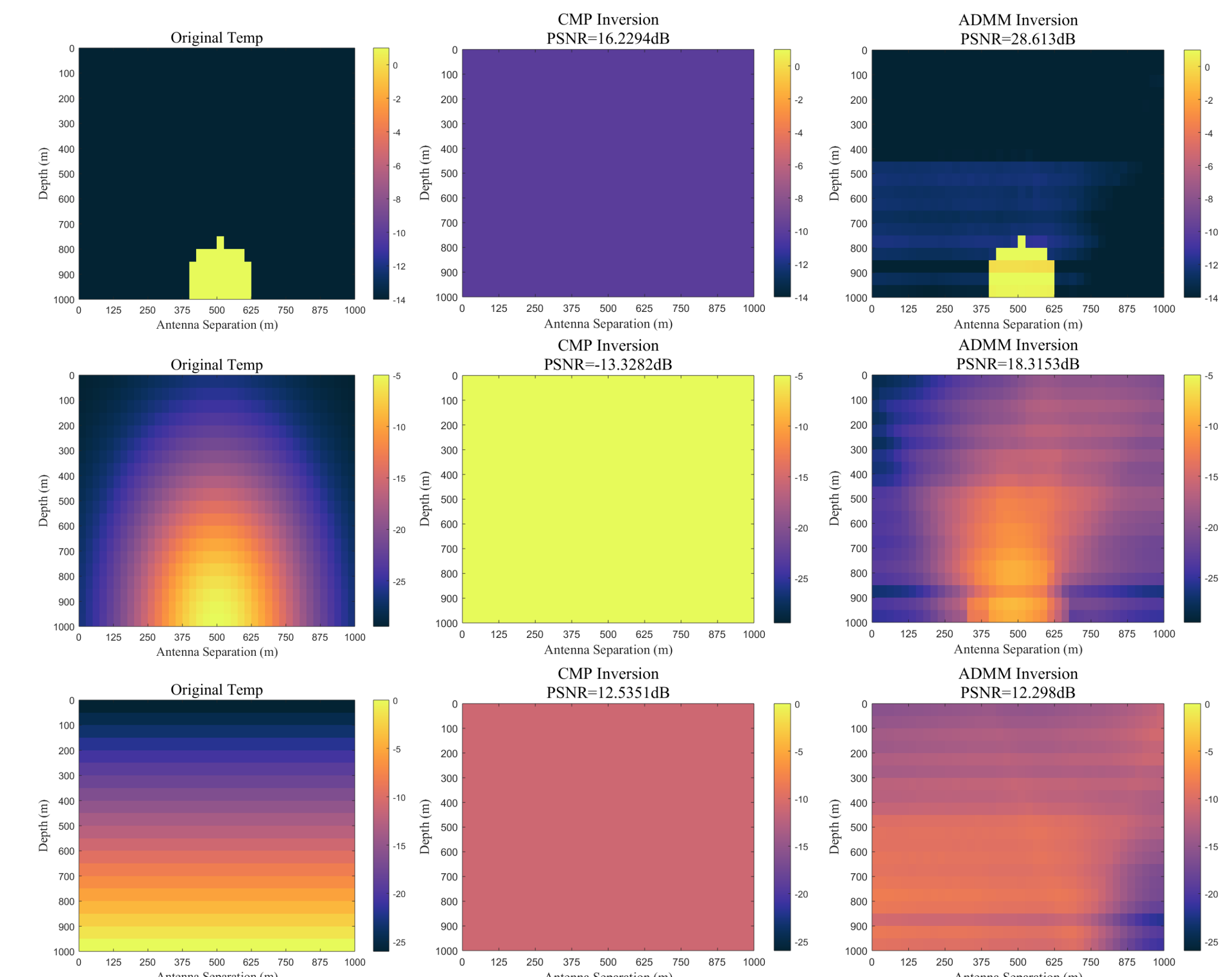
Technique Comparison

Traditional Radar Techniques:
Monostatic

- Estimate depth and along track averaged attenuation/temp
- Use depth averaged attenuation and physics models to estimate temperature distribution

Bistatic

- Measure depth averaged conductivity with a common midpoint (CMP) experiment.



$$\sigma = 2 \sqrt{\frac{\epsilon}{\mu}} \ln \left(\frac{P_{RX} K G(\theta_1) \cos^2(\theta_1) \Gamma(\theta_1)^2}{P_{RX} G(\theta_k) \cos^2(\theta_k) \Gamma(\theta_k)^2} \right) \frac{1}{-2H \left(\frac{1}{\cos(\theta_k)} - \frac{1}{\cos(\theta_1)} \right)} [3]$$