

# ADMM Applied to Bistatic Radar Tomography

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## OBJECTIVE

To apply ADMM and other inversion techniques discussed in class to a tomography problem. Specifically, I want to forward model received radar power for a given material temperature profile and then use ADMM to invert for the material's conductivity distribution to then solve for temperature.

## MOTIVATION & RELATED WORK

Most of Antarctica's contribution to sea level rise originates from fast flowing outlet glaciers and ice streams, but the physical processes controlling ice stream width are poorly understood when topographic controls are absent [1]. Recent modeling studies indicate that ice stream width may be controlled by elevated temperatures along the ice stream's edge, an area called a shear margin [2]. While radio echo sounders can provide measurements of englacial water storage and subglacial conditions, existing radar sounding techniques cannot measure temperature and water distributions to test these hypotheses. Monostatic radar measurements only discern depth averaged attenuation accurately because they don't have enough independent measurements to separate depth dependent attenuation from basal reflectivity and scattering. Bistatic radars could solve this problem by providing sufficient independent measurements to separate the influence of temperature/attenuation, reflectivity and scattering through inverse theory. Bistatic radar has rarely been applied to glaciology; one of the only bistatic studies attempted to infer semi-depth dependent attenuation but it was primarily unsuccessful due to the SNR issues [3]. That study did not use inverse theory, but it has been used in glaciology to solve for uncompacted snow density from EM wave speed in shallow systems [4]. Unfortunately, this method doesn't work on ice sheets where the density is constant. Instead of using wave speed, attenuated radar power can be used to invert for temperature, because warmer ice is more conductive and attenuates the signal more. I would like to create a simple EM simulation to forward model the radar data and see if I can use ADMM and/or other inversion techniques discussed in class to invert for the temperature distribution applied in the EM simulation. This will be the first step in determining if bistatic radar can infer an accurate temperature distribution inside a glacier. If it is possible, then bistatic radar could shed some light on the physics controlling ice stream width and enable more accurate sea level rise estimates.

## THEORY

The power received by the radar is described as:

$$P_{RX} = P_{TX} G_1 G_2 \rho \left( \frac{\lambda}{4\pi r} \right)^2 \Gamma^2 e^{-2\alpha r} \quad (1)$$

Ice is a low loss material, so the attenuation coefficient is:

$$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} \quad (2)$$

The inversion would solve for conductivity which can then be related to temperature with the equation below [5]. The equation accounts for the conductivity contributions of pure ice ( $i=0$ ),  $H^+$  ions ( $i=1$ ), and salt ( $i=2$ ).

$$\sigma = \sum_{i=0}^2 \sigma_0^i C_i e^{-\frac{E_i}{k} \left( \frac{1}{T} - \frac{1}{T_r} \right)} \quad (3)$$

The proposed simulation geometry includes a stationary transmitter and several receivers as depicted in figure 1. For this geometry, we can use ray theory to write the received power in terms of the electrical properties of each grid space and the distance ( $\Delta r_n$ ) the ray travels through each grid space. If we can solve for conductivity, then temperature can be directly determined by assuming the ice composition and applying equation 3. The power at receiver  $k$  is given by:

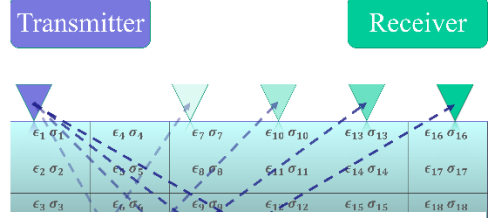


Figure 1: Simulation geometry. There will be one transmitter and several receivers.

$$P_k = P_{TX} G^2(\theta_k) \rho \Gamma_k^2(\theta_k) \left( \frac{\lambda}{4\pi r_k} \right)^2 e^{-\sum_{n=1}^{\# \text{ grids}} \frac{\sigma_n}{2} \sqrt{\frac{\mu}{\epsilon_n}} \Delta r_{k,n}}$$

The transmitted power, antenna gain, scattering coefficient, and permeability are known. The conductivity and reflection coefficient vary significantly, and the permittivity can change from roughly 3.14 to 3.22. The permittivity is independent of temperature, and literature typically treats permittivity as constant, so I will do the same. For a first attempt, I will treat the reflection coefficient as being known, because if it is unknown, I am unsure how to frame the problem in the  $\min ||Cx - b||_2^2$  format. With these assumptions, the only variables dependent on the grid number ( $n$ ) are the conductivity and distance the ray travels through that grid ( $\Delta r_n$ ). We can rewrite the equation above to separate the variables dependent on the grid number ( $n$ ) and the receiver's number ( $k$ ):

$$\frac{-2 \ln(P_k)}{\sqrt{\frac{\mu}{\epsilon}} \ln \left( P_{TX} G^2(\theta_k) \rho \Gamma_k^2(\theta_k) \left( \frac{\lambda}{4\pi r_k} \right)^2 \right)} = \sum_{n=1}^{\# \text{ grids}} \sigma_n \Delta r_{k,n}$$

Letting the left side of the equation be  $b_k$ , then we can use the  $Cx - b$  format. We can add weight to keeping the  $\sigma_n$  smooth with a differentiation matrix applied to  $\sigma_n$  written as  $\lambda |z|_1$ . The objective function is then:

$$\text{Minimize} \left\| \begin{bmatrix} \Delta r_{1,1} & \cdots & \Delta r_{1,n} \\ \vdots & \ddots & \vdots \\ \Delta r_{k,1} & \cdots & \Delta r_{k,n} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \right\|_2^2 - \lambda |z|_1$$

This minimization problem can then be solved with ADMM.

#### TIMELINE:

- Week 1: Develop a better understanding of inverse theory and better formulate the objective function and inverse approach. Read lecture 10 notes and “Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers.”
- Week 2: Finish forward model for simulating received power given a temperature profile.
- Week 3: Write ADMM code to invert for temperature profile.
- Week 4: Investigate robustness of inversion and look into other inversion methods.

#### REFERENCES

- [1] Andrew Shepherd et al., “Trends and connections across the antarctic cryosphere,” Nature, vol. 558, no. 7709, pp.223, 2018.
- [2] Jenny Suckale et al., “Deformation-induced melting in the margins of the west antarctic ice streams,” J. Geophys.Res. Earth Surf., vol. 119, no. 5, pp. 1004–1025, 2014.
- [3] Nicholas Holschuh et al., “Constraining attenuation uncertainty in common midpoint radar surveys of ice sheets,” J. Geophys. Res. Earth Surf., vol. 121, no. 10, pp. 1876–1890, 2016.
- [4] R. Drews, et al., “Constraining variable density of ice shelves using wide-angle radar measurements,” Cryosphere, 10(2), pp. 811–823, 2016.
- [5] Joseph A MacGregor et al., “Modeling englacial radar attenuation at siple dome, west antarctica, using ice chemistry and temperature data,” J. Geophys. Res. Earth Surf., vol. 112, no. F3, 2007.

| TABLE OF VARIABLES                                    |                                                                                                    |
|-------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| Equation 1 variables                                  |                                                                                                    |
| $P_{RX}$                                              | Received power                                                                                     |
| $P_{TX}$                                              | Transmitted power                                                                                  |
| $G$                                                   | Antenna gain                                                                                       |
| $\rho$                                                | Scattering ( $\sigma$ in traditional literature)                                                   |
| $r$                                                   | Distance from radar to reflection point                                                            |
| $\Gamma$                                              | Reflection coefficient                                                                             |
| $\alpha$                                              | Attenuation coefficient                                                                            |
| $\lambda$                                             | Wavelength                                                                                         |
| Equation 2 variables                                  |                                                                                                    |
| $\epsilon = \epsilon_{eff} = \epsilon' + j\epsilon''$ | Permittivity (how much the electric field decreases in a medium)                                   |
| $\mu$                                                 | Permeability (how much the magnetic field changes in a medium like iron)                           |
| $\sigma$                                              | Conductivity                                                                                       |
| Equation 3 variables                                  |                                                                                                    |
| $\sigma^0$                                            | Molar conductivity<br>$\sigma_0^0 = 9.2\mu S/m, \sigma_1^0 = 3.2 S/m/M, \sigma_2^0 = 0.43 S/m/M$ . |
| $C_i$                                                 | Concentration of material in units of mol/L                                                        |
| $E$                                                   | Activation energy: $E_0=0.51\text{eV}, E_1=0.2\text{eV}, E_2=0.19\text{eV}$                        |
| $k$                                                   | Boltzmann's constant: $8.617 \times 10^{-6} \text{eV/K}$                                           |
| $T_r$                                                 | Reference temperature at which the activation energies were measured: 251K                         |
| $T$                                                   | Temperature of the ice in kelvin                                                                   |