

Denoising of Computed Tomography Scans

Neal Patel
Stanford University
450 Serra Mall, Stanford, CA 94305
nealpatel@stanford.edu

Perry Alagappan
Stanford University
450 Serra Mall, Stanford, CA 94305
palagap@stanford.edu

Abstract

ADMM Denoising was computed with three priors (TV, NLM, and BM3D) for three types of CT images (brain, lungs, and abdomen). For two noise types – Gaussian and Poisson – three noise parameters were used: for Gaussian, $\sigma = 3, 5, 7$ and for Poisson $\lambda = 5, 10, 20$. Results indicated that PSNR values ranged between ~ 20 dB and ~ 30 dB. In both cases, lower noise corresponded to higher PSNR and lower residual. Across all images, Poisson-denoising performed better than Gaussian-denoising. Moreover, the images with lungs had the greatest PSNRs compared to those of the brain and lungs. Additionally, NLM-prior denoising performed best compared to TV and BM3D priors.

1. Introduction

Computed Tomography (CT) is a method used to generate cross-sectional images of a patient's body. Specifically, it uses X-Rays that are beamed at and rotated around the patient's body to generate a cross-sectional "slice". Slices can provide insight into disease or injury; in particular, CT imaging has proven effective at detecting tumors or lesions in the abdomen and pelvic regions.^[1]

One of the critical problems in CT imaging is the imaging of bariatric patients. These patients typically have a body-mass index (BMI) that is greater than 30. CT imaging of bariatric patients often contain large amounts of noise. This occurs because the additional adipose tissue of patients results in attenuated X-Ray beams, which lowers the overall quality of the image. In order for radiologists to be able to provide an accurate diagnosis of medical conditions in such cases, the noise in scans must be mitigated.^{[2][3]}

In the United States, obesity has become widespread. As Figure 1 illustrates, the obesity rate has increased rapidly within two decades. This study, however, is also applicable to non-bariatric patients. To ensure the safety of the patient, it is best to conduct CT imaging with limited radiation exposure. Though this will result in noisier images, the ability to denoise images ensures that diagnoses of health issues can remain accurate despite lesser radiation exposure.^[2]

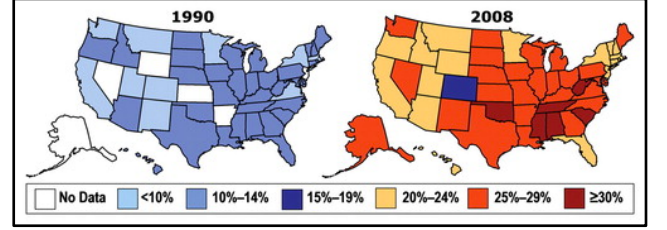


Figure 1: Obesity Prevalence by State in the U.S.

Our study is also motivated by the recent spread of Coronavirus Disease 2019 (COVID-19). Images of patients infected by the virus reveal ground-glass opacities (GGO), and vascular enlargement in the lesion of the lungs. Effective CT imaging procedures can not only help healthcare providers determine whether patients are affected by the disease, but can also provide insight into its effects on the respiratory system.^[4]

1.1. The Radon Transform

A concept critical to our study is the radon transform. The radon transform is an integral transform that maps a function from the spatial domain to the radon or tomographic domain. This plane represents the projection data obtained as the output of a tomographic scan. It is known as a sinogram because the Radon transform of an off-center point source is a sinusoid. The Radon transform is defined as follows:

$$Rf(L) = \int_L f(\mathbf{x}) |d\mathbf{x}|.$$

Equation 1: The Radon Transform

Where $f(\mathbf{x})$ is a function of two variables s.t. $f(\mathbf{x}) = f(x,y)$.

The Shepp-Logan Phantom, a standard test image created by Larry Shepp and Benjamin F. Logan for their paper *The Fourier Reconstruction of a Head Section*, is illustrated below. It serves as a model of a human head in the development and testing of image reconstruction algorithms. Figure 2 illustrates the image after it is radon-transformed into the tomographic domain, while Figure 3 illustrates the image after the radon-transformed image is inverse radon-transformed back into the spatial domain. One should note that the inverse radon-transformed image does not perfectly resemble the original image.^[5]

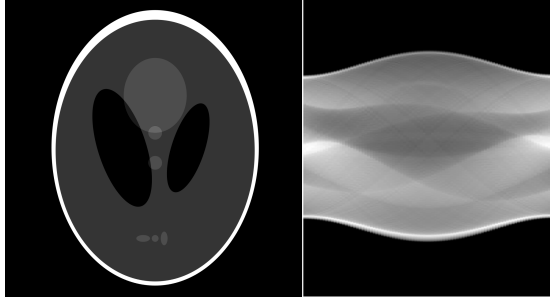


Figure 2: Schepp Logan Original Image & Radon-Transformed CT Image

1.2. Additive Noise

In this study, we will inject two types of noise into the radon-transformed CT image. The first, is Gaussian noise, which follows the Gaussian distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Equation 2: The Gaussian Distribution

And the second is Poisson noise, which follows the Poisson distribution:

$$f(k; \lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Equation 3: The Poisson Distribution

Previous studies indicate that Poisson noise is the most likely noise to be distributed in a CT image. However, for comparison, we will also inject Gaussian noise into a CT image. [3]

2. Related Work

Table 1 summarizes some of the existing techniques used for denoising of CT images, along with their advantages and disadvantages: [6]

Commonly Used Noise Reduction Methods			
Description	Sample Methods	Advantages	Disadvantages
Image space de-noising	SafeCT, SharpView, aNLM	Fast, needs only reconstructed images	Does not incorporate system physics
Projection space denoising	Adaptive filtering or iterative denoising in projection data	Fast, may incorporate complex system physics	Potential loss of spatial resolution if not designed well

Hybrid iterative Reconstruction	SAFIRE	Speedy noise reduction, artifact reduction by iteration	May change the noise texture of CT images
Full iterative reconstruction	MBIR	Incorporates system model, reduces both noise and artifacts	Slow, may change the noise texture of image

Table 1: Commonly used noise-reduction techniques

Our objective was to combine an iterative image reconstruction method with an image prior. While research indicated that studies had been done in iterative reconstruction methods and some denoising techniques we were familiar with (e.g. Non-local means), we did not find any studies that combined the two methodologies.

3. Methodology

To solve the problem of image reconstruction, we chose the method of Alternating Direction Method of Multipliers (ADMM). We used three priors to reconstruct the original image: a Total Variation (TV) Prior, a Non-Local Means (NLM) prior, and a Block Matching and 3D Filtering (BM3D) Prior. We tested three CT scan images: of the abdominopelvic cavity, the brain, and the lungs. In each case, we tested ADMM reconstruction for each of the three priors and two types of noise, Gaussian and Poisson. Though Poisson noise is generally the form of noise that appears in CT images, we also tested Gaussian for reference. Three levels of noise were tested for each scenario: for Gaussian, $\sigma = 3, 5, 7$ and for Poisson $\lambda = 5, 10, 20$. In each case, we took a spatial image, converted it to the radon domain, injected the noise into that image, and then used ADMM to denoise that image in the spatial domain. The major techniques we used are detailed below.

3.1. ADMM

The alternating direction method of multipliers (ADMM) is an algorithm that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle. In this case, ADMM splits an objective into two weighted sums: one for the image, and one known as the regularization term:

$$\begin{aligned} &\underset{\{x\}}{\text{minimize}} && \underbrace{\frac{1}{2}\|b - Ax\|_2^2}_{f(x)} + \underbrace{\lambda\Gamma(z)}_{g(z)} \\ &\text{subject to} && Kx - z = 0 \end{aligned}$$

Equation 4: ADMM Setup

3.2. TV Prior

The TV prior computes the gradients of the original image in both the X and Y directions. Hence, images with sparse gradients (clearly defined edges), are more suited to TV prior denoising. The equation below demonstrates the calculation of a TV prior based on an input image, x .

$$TV(x) = \|\nabla_x x\|_1 + \|\nabla_y x\|_1 = \left\| \begin{bmatrix} \nabla_x \\ \nabla_y \end{bmatrix} x \right\|_1$$

Equation 5: Constructing a TV Prior

3.3. NLM Prior

Non-local means averages pixels based on neighboring pixel patches that are similar to them. However, these pixels do not need to be clustered around the target pixel; they can be from any part of the image. Thus, NLM is a robust self-similarity prior. NLM is defined as follows:

$$i_{denoised}(x) = \frac{1}{\sum_{\text{all pixels } x'} w(x, x')} \sum_{\text{all pixels } x'} i_{noisy}(x') \cdot w(x, x')$$

$$w(x, x') = \exp \left(- \frac{\|W(i_{noisy}, x') - W(i_{noisy}, x)\|^2}{2\sigma^2} \right)$$

Equation 6: Constructing an NLM Prior

We used an existing NLM implementation by Dirk Jan-Kroon and integrated it with our denoising ADMM program.

3.4. BM3D Prior

Like NLM, BM3D selects patches of an image to denoise a target region. It groups them into 3D blocks and then applies a collaborative filter as follows:

- DCT-transform each 3D block
- Threshold transform coefficients
- Inverse transform 3D block

We used an existing BM3D package by Makinen et al, and integrated it with our denoising ADMM program.

4. Results

Here we detail our results for the three images used, for each of two noise types and three noise levels. Please note that for brevity, we have only displayed selected results. The following are the original images we use: [7] [8] [9]

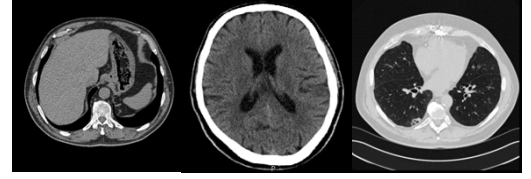


Figure 3: Images of Abdomen, Brain, and Lungs

4.1. Reconstructed Images

4.1.1. Abdomen Images

4.1.1.1. Gaussian Noise

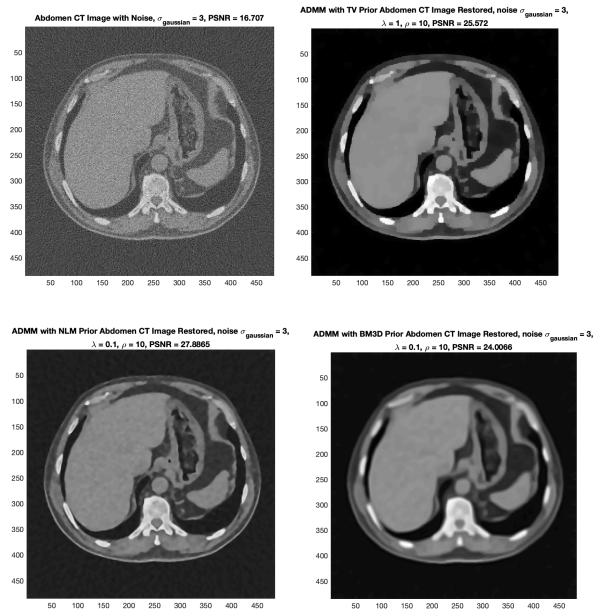
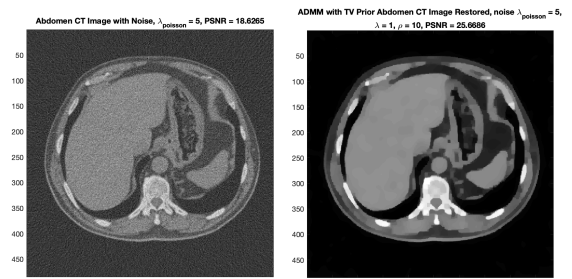


Figure 4: Abdomen Gaussian Reconstruction

4.1.1.2 Poisson Noise



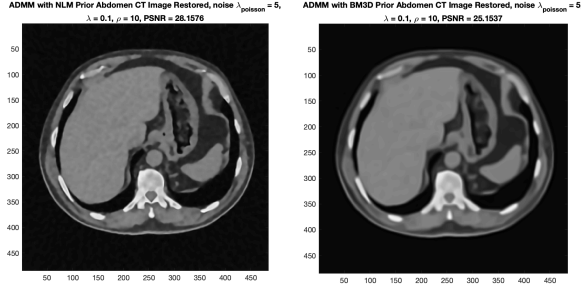


Figure 5: Abdomen Poisson Reconstruction

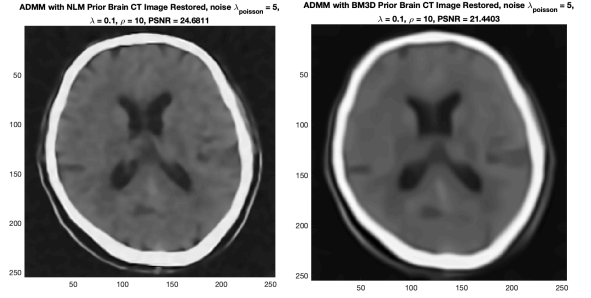


Figure 7: Brain Poisson Reconstruction

4.1.2. Brain Images

4.1.2.1. Gaussian Noise

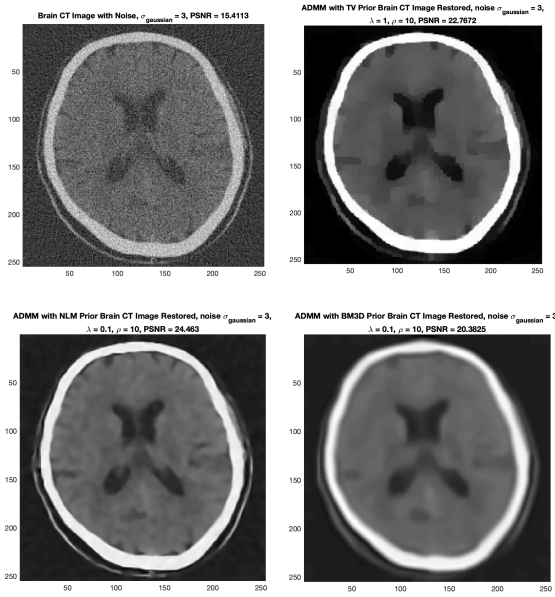


Figure 6: Brain Gaussian Reconstruction

4.1.3. Lung Images

4.1.3.1. Gaussian Noise

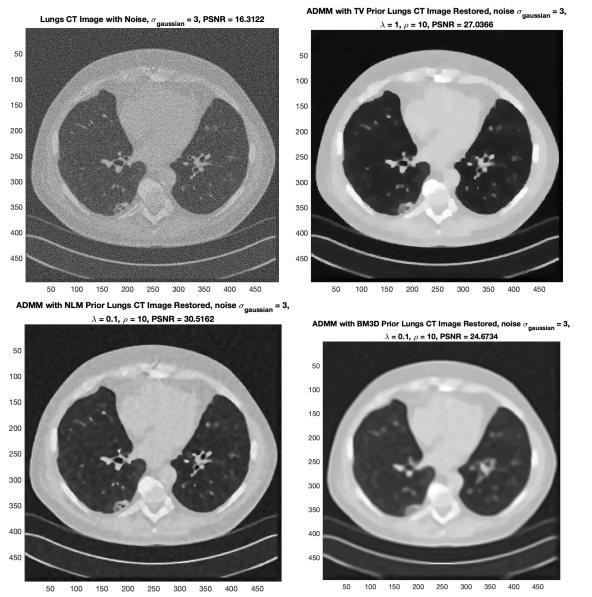
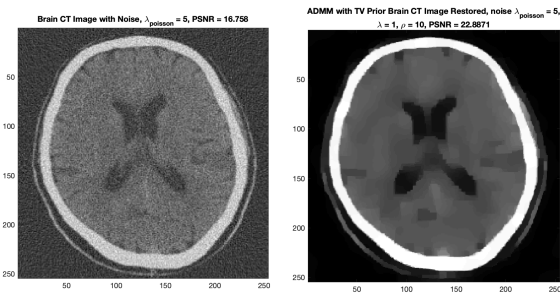
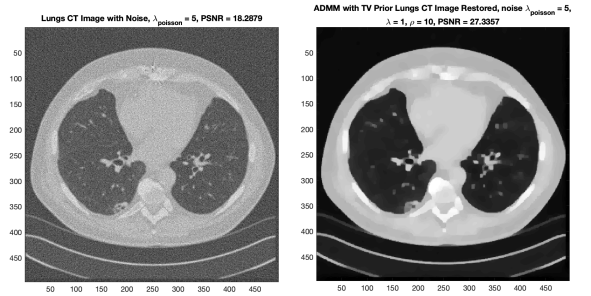


Figure 8: Lungs Gaussian Reconstruction

4.1.2.2 Poisson Noise



4.1.3.2 Poisson Noise



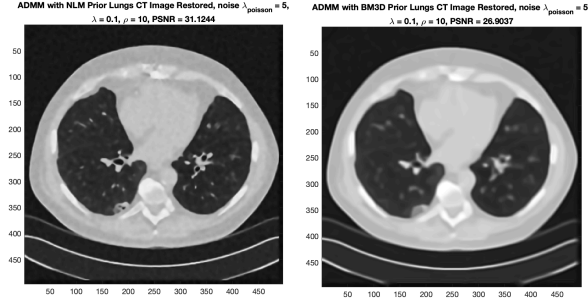


Figure 9: Lungs Poisson Reconstruction

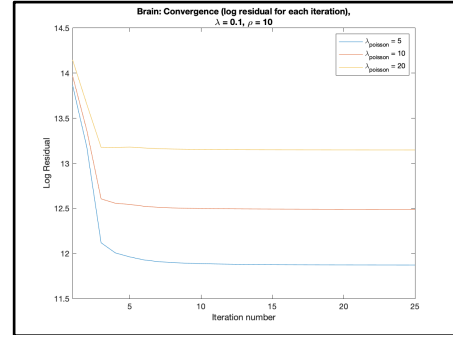


Figure 11: Brain PSNR and Residual Plots

4.2. Residual Plots (NLM, Poisson)

4.2.1. Abdomen

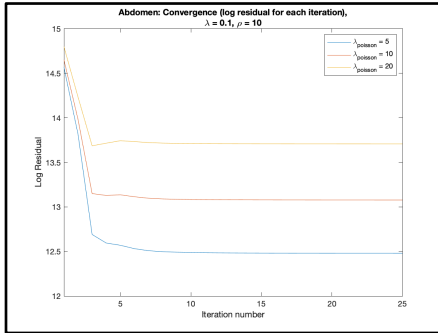
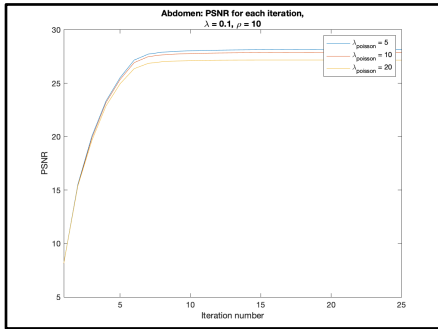


Figure 10: Abdomen PSNR and Residual Plots

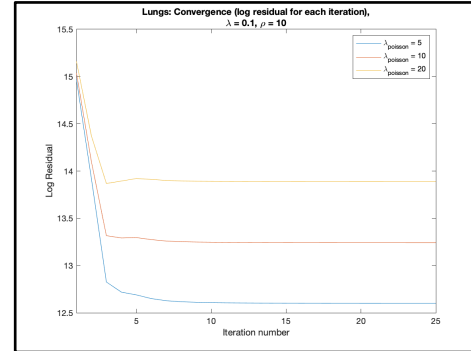
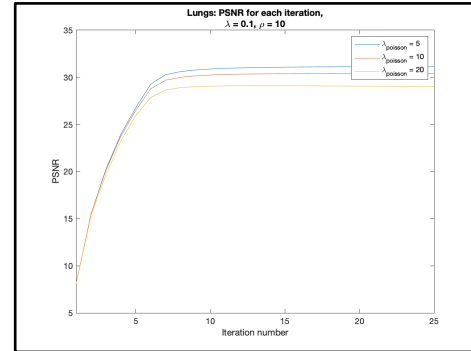
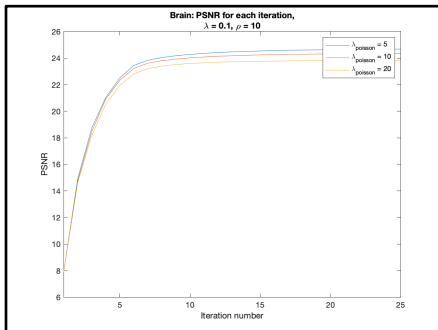


Figure 12: Lungs PSNR and Residual Plots

4.2.2. Brain



Denoised Image PSNR Values (dB), $\sigma = 3, \lambda = 5$						
	TV		NLM		BM3D	
	G	P	G	P	G	P
Abdomen	25.6	25.7	27.9	28.2	24.0	25.2
Brain	22.8	22.9	24.5	24.7	20.4	21.4
Lungs	27.0	27.3	30.5	31.1	24.7	26.9

Table 2: Gaussian vs Poisson Noise PSNR Results

5. Analysis

As seen from Table 2, generally, we were able to reconstruct images with Poisson noise better than we were able to reconstruct images with Gaussian noise. This is logical, given that the noisy Poisson images all have higher PSNRs than the noisy Gaussian images. However, we did not calibrate the Poisson and Gaussian noise levels to the same reference; we simply chose reasonable noise values. This could explain the enhanced performance of Poisson-denoising over Gaussian-denoising.

Looking at the PSNR graphs, we also see that for both Poisson and Gaussian noise, a larger noise value corresponds to a smaller PSNR. Likewise, for both types of noise, larger noise corresponds to a larger residual. These are expected results.

Comparing the overall denoising between the three types of images – abdomen, brain, and lungs – we find that the lungs-denoising performed best while the brain-denoising performed the worst. Examining the images, we can see one potential reason why this is so. The lungs image has a very clear contrast between the lungs and the thoracic cavity (area surrounding it). On the other hand, the brain image doesn't have a significant contrast within the white border. Our denoising techniques perform best when there is clear demarcation between different edges or components of the image.

Finally, we found that the NLM prior performed best compared to TV and BM3D priors for denoising. After examining the images, we see that this is because NLM maintains granular features of the image. Denoising with TV and BM3D on the other hand creates blocky regions of the resulting images, decreasing their quality.

6. Conclusion

We were able to successfully denoise noisy CT images of the brain, lungs, and abdomen. PSNRs for all of the resulting images ranged between ~20 dB and ~30 dB. Of the three types of images, we found that we were able to denoise the lungs best. Additionally, the ADMM + NLM pairing performed best. It is expected that NLM performs better than TV; BM3D performed about as well as TV. Finally, as expected, we found that PSNR increased as iteration increased and the residuals decreased as the iteration number increased. One critical finding is that NLM is the best technique for CT denoising, as the reconstructed image preserves texture of the original image, enabling detection of lesions. With TV and BM3D reconstruction, the reconstructed image is blockier, making it harder to detect irregularities.

One limitation of our study is how specialized ADMM reconstruction is. Optimal denoising of the images we have selected will require a more expansive, flexible framework that can incorporate any denoising algorithm. Regularization by Denoising (RED) is one such framework that could enhance denoising of CT images. ^[10]

Acknowledgements

We would like to sincerely thank Gordon Wetzstein, Chris Metzler, and David Lindell for their gracious help on this project and throughout the course.

References

- [1] "Computed Tomography (CT)." National Institute of Biomedical Imaging and Bioengineering, U.S. Department of Health and Human Services, www.nibib.nih.gov/science-education/science-topics/computed-tomography-ct.
- [2] Modica, Michael J., et al. "The Obese Emergency Patient: Imaging Challenges and Solutions." *RadioGraphics*, 4 May 2011, pubs.rsna.org/doi/full/10.1148/rg.313105138.
- [3] Wang, Jing, et al. "An Experimental Study on the Noise Properties of X-Ray CT Sinogram Data in Radon Space." *Physics in Medicine and Biology*, U.S. National Library of Medicine, 21 June 2008.
- [4] Kent, Chloe. "CT Scans of Coronavirus Patients Reveal Nature and Extent of Lung Damage." *Verdict Medical Devices*, 4 Mar. 2020, www.medicaldevice-network.com/news/coronavirus-ct-scans/.
- [5] Ellenberg, Jordan (February 22, 2010). "Fill in the Blanks: Using Math to Turn Lo-Res Datasets Into Hi-Res Samples". *Wired*. 12 March 2020.
- [6] Methods for Clinical Evaluation of Noise Reduction Techniques in Abdominopelvic CT. Ehman et. al. <https://pubs.rsna.org/doi/pdf/10.1148/rg.344135128>
- [7] "Abdomen CT Scan." *Insight Medical Imaging*, 7 Jan. 2020, x-ray.ca/services/computed-tomography/abdomen-ct-scan/.
- [8] Orenstein, David. "CT Lung Screening Appears Cost-Effective." *CT Lung Screening Appears Cost-Effective | News from Brown*, 5 Nov. 2014, news.brown.edu/articles/2014/11/cancercost.
- [9] Naqi, Rohana & Azeemuddin, Muhammad. (2013). Naeglaeria infection of the central nervous system, CT scan findings: A case series. *Journal of the Pakistan Medical Association*. 63. 399 - 402.
- [10] Romano, et al. "The Little Engine That Could: Regularization by Denoising (RED)." *ArXiv.org*, 3 Sept. 2017, arxiv.org/abs/1611.02862.