Analysis of Richardson-Lucy Deconvolution with a TV Prior

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RL+TV_A
(incorrect)

$$\nabla\lambda\|\mathbf{Dx}\|_1 = \lambda\nabla\sum_{ij}\left(\sqrt{(\nabla_x\mathbf{x})_{ij}^2}+\sqrt{(\nabla_y\,\mathbf{x})_{ij}^2}\right) = -\lambda\left(\frac{\mathbf{D}_x\,\mathbf{x}}{|\mathbf{D}_x\,\mathbf{x}|}+\frac{\mathbf{D}_y\,\mathbf{x}}{|\mathbf{D}_y\,\mathbf{x}|}\right)$$

$$\mathbf{x}^{(q+1)} = \frac{\mathbf{A}^T\left(\frac{\mathbf{b}}{\mathbf{Ax}}\right)}{\mathbf{A}^T\mathbf{1}-\lambda\left(\frac{\mathbf{D}_x\,\mathbf{x}}{|\mathbf{D}_x\,\mathbf{x}|}+\frac{\mathbf{D}_y\,\mathbf{x}}{|\mathbf{D}_y\,\mathbf{x}|}\right)}\cdot\mathbf{x}^{(q)}$$

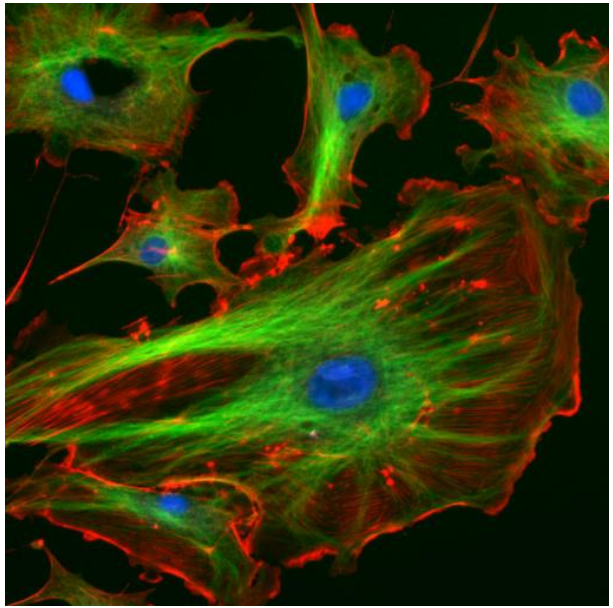
RL+TV_B
(correct)

$$\nabla\lambda\|\mathbf{Dx}\|_1 = \lambda\nabla\sum_{ij}\left(\sqrt{(\nabla_x\mathbf{x})_{ij}^2}+\sqrt{(\nabla_y\,\mathbf{x})_{ij}^2}\right) = \lambda\left(\mathbf{D}_x^T\frac{\mathbf{D}_x\,\mathbf{x}}{|\mathbf{D}_x\,\mathbf{x}|}+\mathbf{D}_y^T\frac{\mathbf{D}_y\,\mathbf{x}}{|\mathbf{D}_y\,\mathbf{x}|}\right)$$

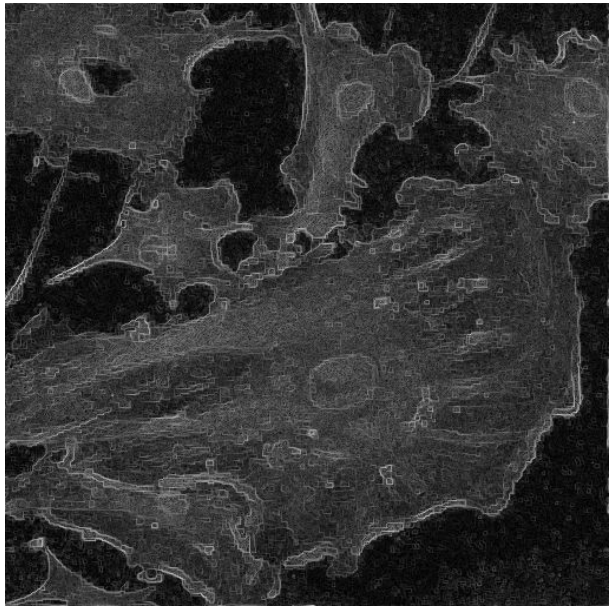
$$\mathbf{x}^{(q+1)} = \frac{\mathbf{A}^T\left(\frac{\mathbf{b}}{\mathbf{Ax}}\right)}{\mathbf{A}^T\mathbf{1}+\lambda\left(\mathbf{D}_x^T\frac{\mathbf{D}_x\,\mathbf{x}}{|\mathbf{D}_x\,\mathbf{x}|}+\mathbf{D}_y^T\frac{\mathbf{D}_y\,\mathbf{x}}{|\mathbf{D}_y\,\mathbf{x}|}\right)}\cdot\mathbf{x}^{(q)}$$

Gradient Magnitude, 100 iterations, λ=0.04

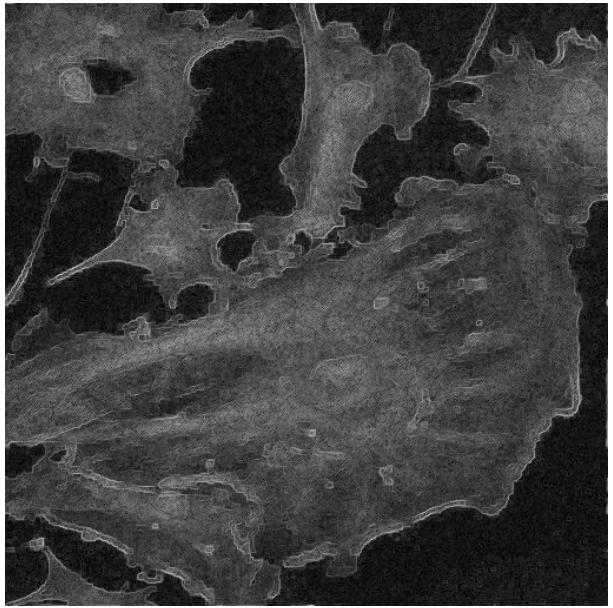
Ground Truth Image



RL+TV_A

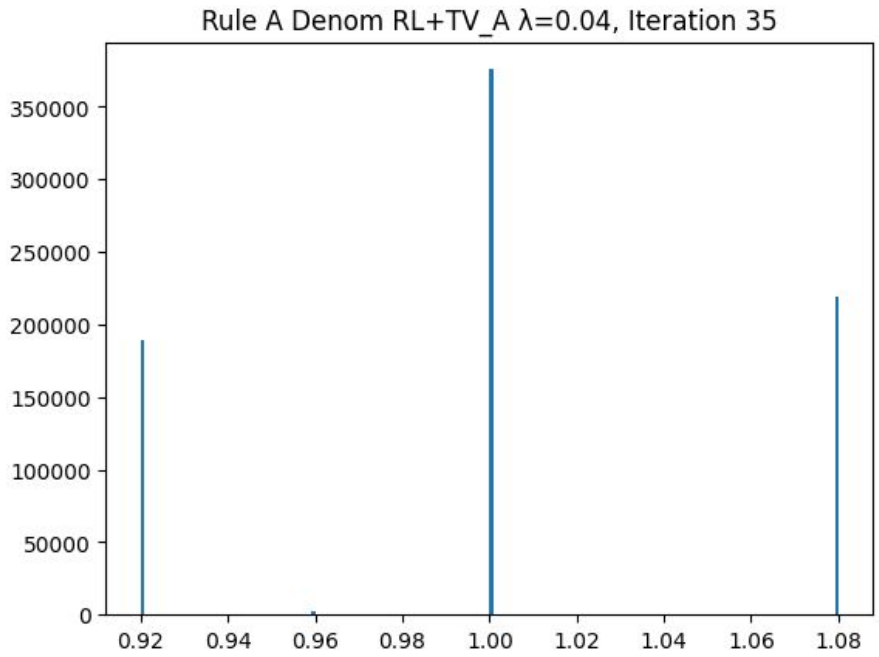


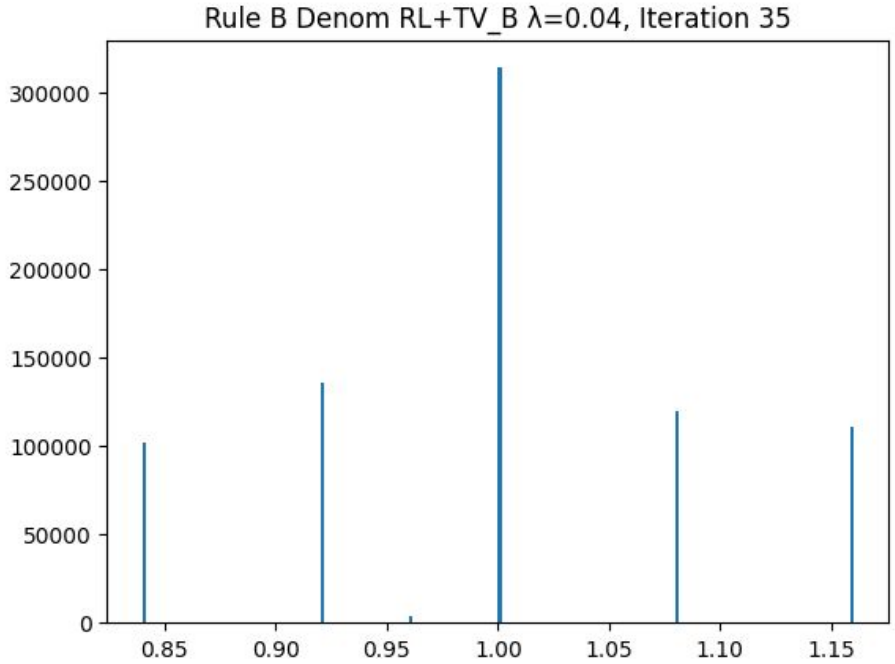
RL+TV_B



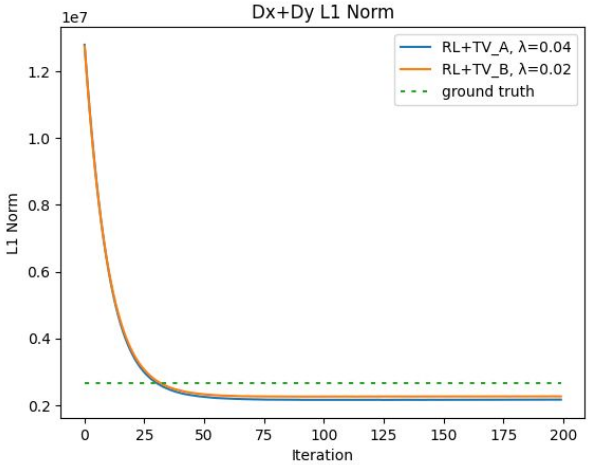
$$\mathbf{A}^T\mathbf{1} - \lambda\left(\frac{\mathbf{D}_x\,\mathbf{x}}{|\mathbf{D}_x\,\mathbf{x}|} + \frac{\mathbf{D}_y\,\mathbf{x}}{|\mathbf{D}_y\,\mathbf{x}|}\right)$$

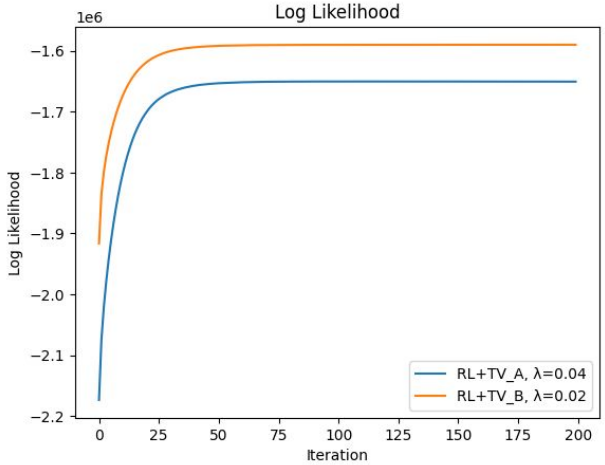
$$\mathbf{A}^T\mathbf{1} + \lambda\left(\mathbf{D}_x^T\frac{\mathbf{D}_x\,\mathbf{x}}{|\mathbf{D}_x\,\mathbf{x}|} + \mathbf{D}_y^T\frac{\mathbf{D}_y\,\mathbf{x}}{|\mathbf{D}_y\,\mathbf{x}|}\right)$$

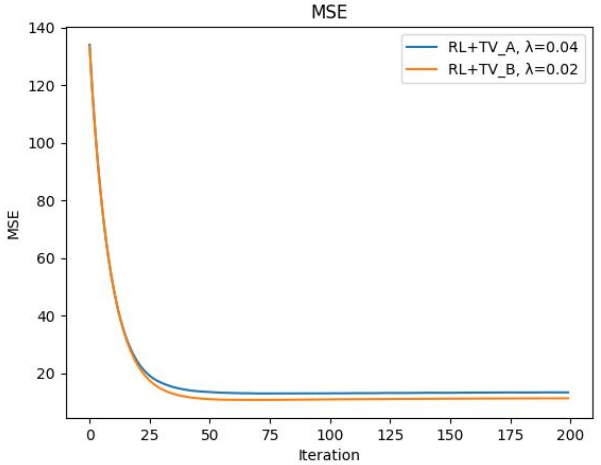




Comparison of RL+TV_A,λ=0.04 & RL+TV_B,λ=0.02

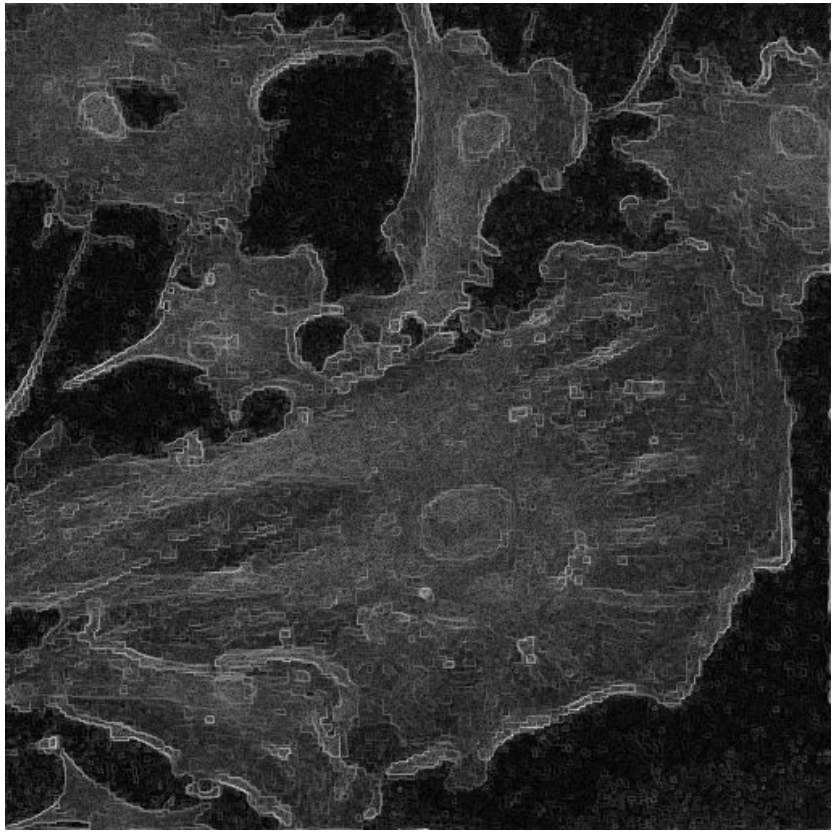




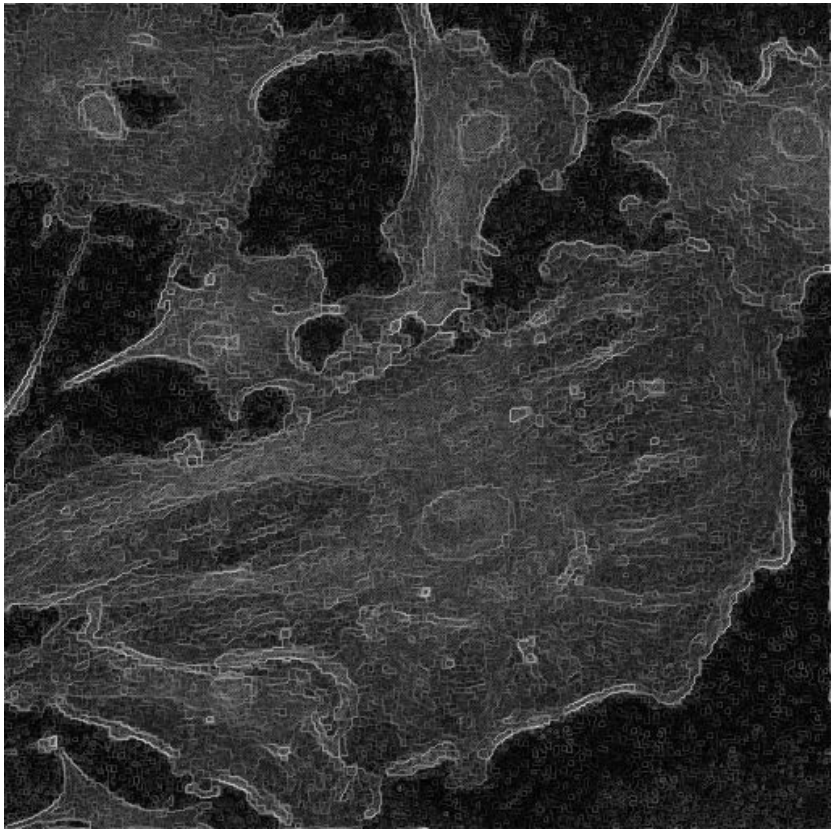


Gradient Magnitude, 200 iterations

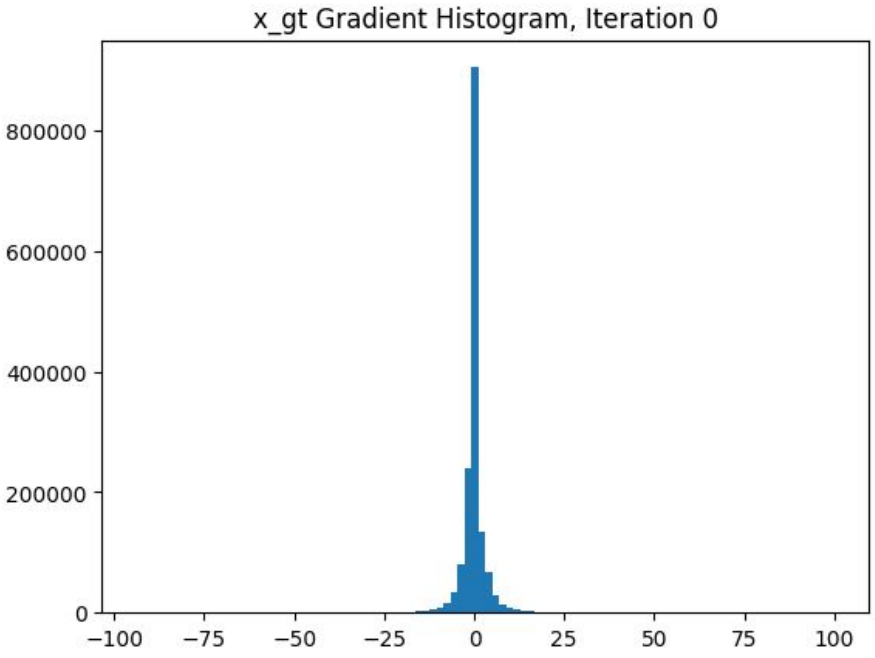
RL+TV_A, ƛ=0.04

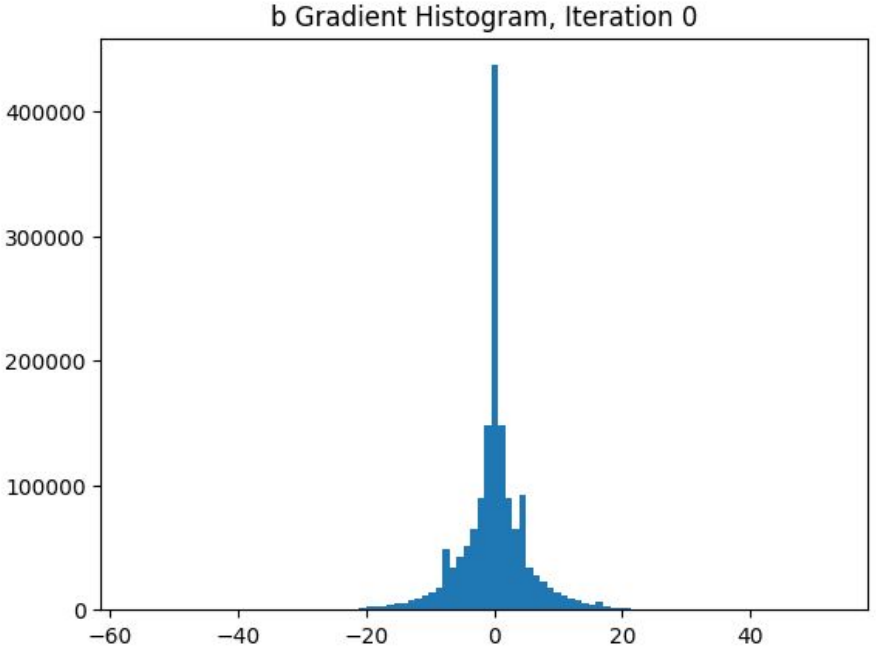


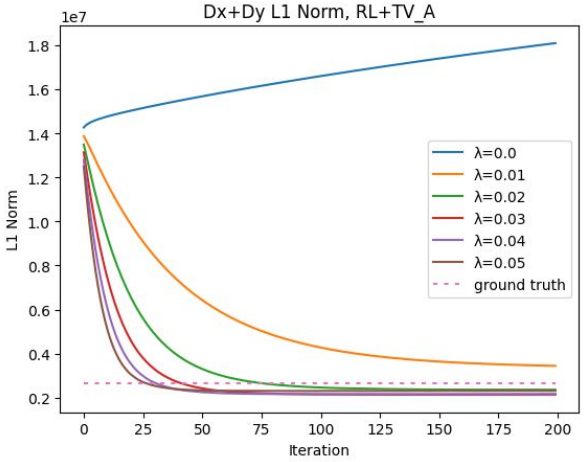
RL+TV_B, ƛ=0.02

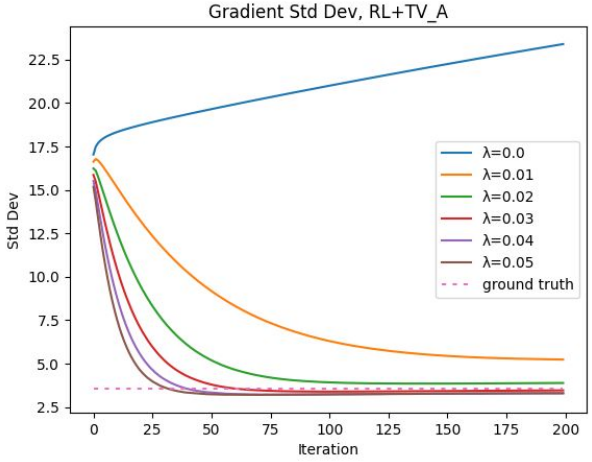


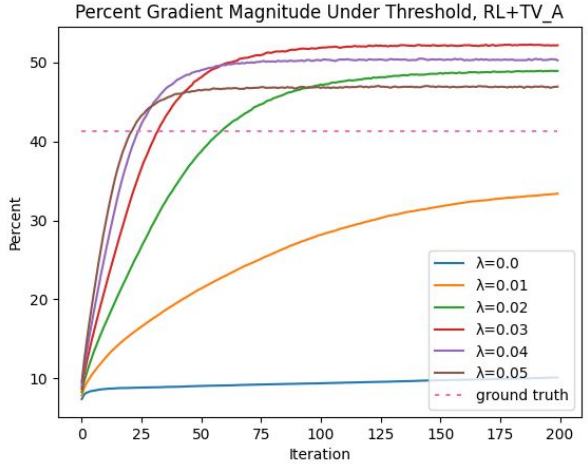
Use Standard Deviation of Gradient Histogram as Measure of Sparsity

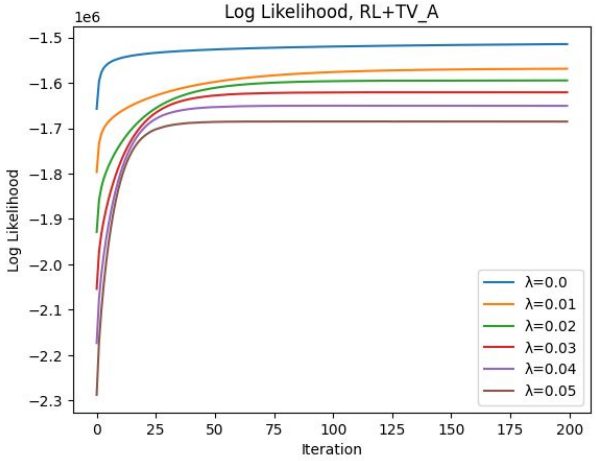


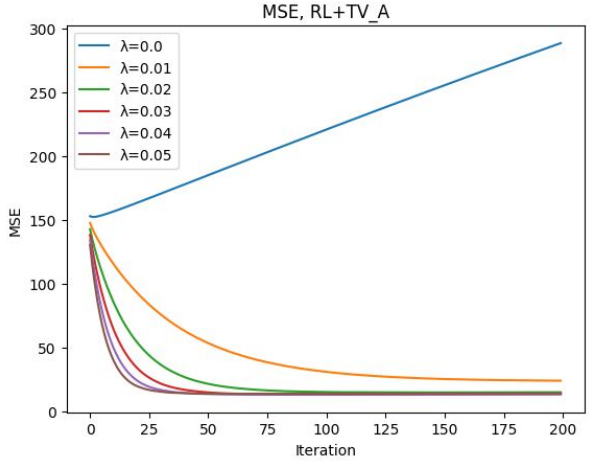


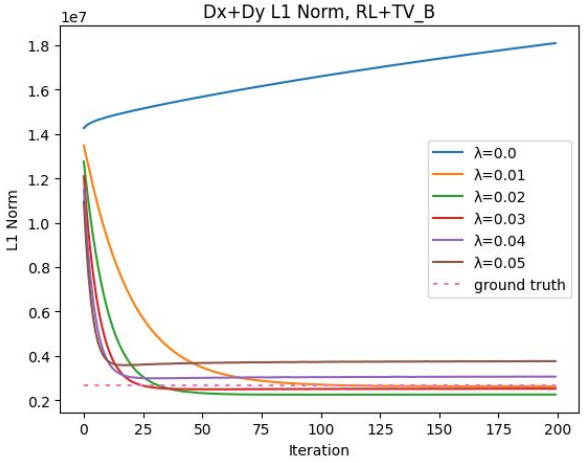

Dx+Dy L1 Norm, RL+TV_A
1e7
L1 Norm
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05
ground truth

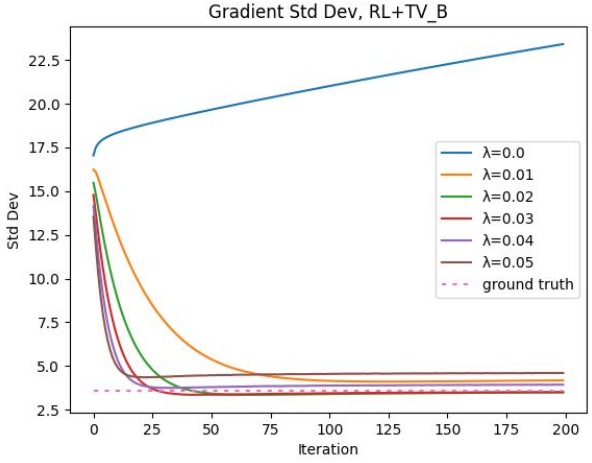

Gradient Std Dev, RL+TV_A
Std Dev
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05
ground truth

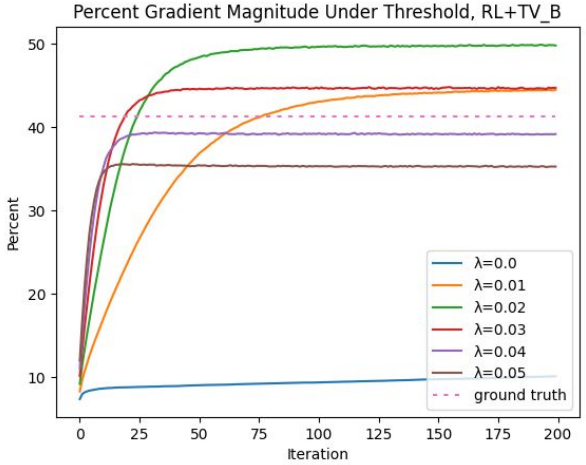

Percent Gradient Magnitude Under Threshold, RL+TV_A
Percent
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05
ground truth

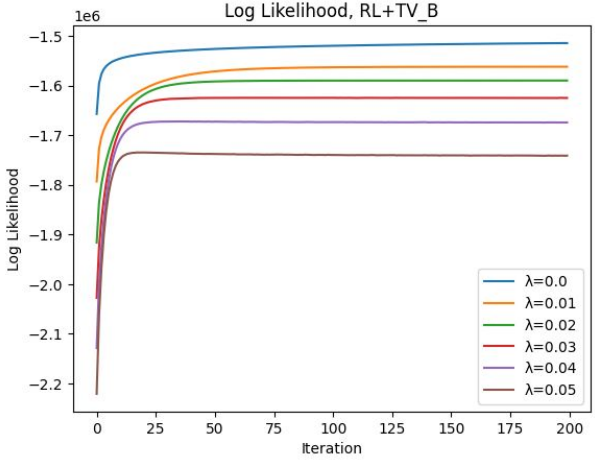

Log Likelihood, RL+TV_A
1e6
Log Likelihood
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05

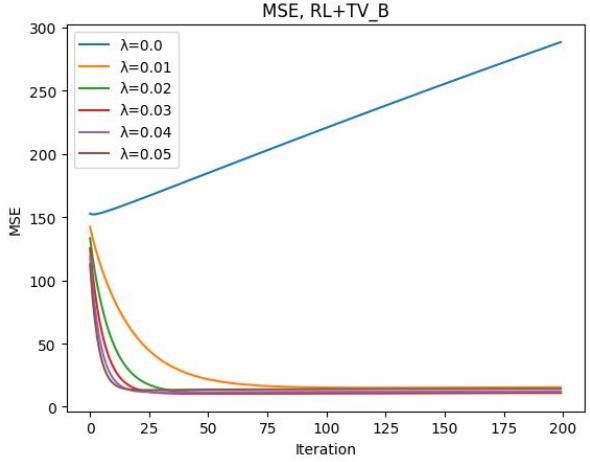

MSE, RL+TV_A
MSE
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05


Dx+Dy L1 Norm, RL+TV_B
1e7
L1 Norm
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05
ground truth


Gradient Std Dev, RL+TV_B
Std Dev
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05
ground truth


Percent Gradient Magnitude Under Threshold, RL+TV_B
Percent
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05
ground truth


Log Likelihood, RL+TV_B
1e6
Log Likelihood
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05


MSE, RL+TV_B
MSE
Iteration
λ=0.0
λ=0.01
λ=0.02
λ=0.03
λ=0.04
λ=0.05

Conclusions

- For all images and the range of λ tested, at the optimal per image λ
  - RL+TV_A produces sparser gradients by all three measures
  - RL+TV_B produces higher log likelihood
  - RL+TV_B usually produces lower MSE

References

1. Gordon Wetzstein, EE 367 / CS 448I Computational Imaging and Display Notes: Noise, Denoising, and Image Reconstruction with Noise (lecture 10) http://stanford.edu/class/ee367/reading/lecture10_notes.pdf

2. Dey, N., Blanc-Feraud, L., Zimmer, C., Kam, Z., Olivo-Marin, J.-C., AND Zerubia, J. 2004. 3d microscopy deconvolution using Richardson-Lucy algorithm with total variation regularization. INRIA Technical Report.

3. Dey, N., Blanc-Feraud, L., Zimmer, C., Kam, Z., Olivo-Marin, J.-C., AND Zerubia, J. 2004. A deconvolution method for confocal microscopy with total variation regularization. In IEEE Symposium on Biomedical Imaging: Nano to Macro, 1223–1226.