# Meta-learning For Solving Inverse Problem

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## Motivation

- The problem of reconstructing $x$ from observation $y$ is called inverse problem.

$$y = A(x) + \epsilon$$

$A(\cdot)$:image formation process; $\epsilon$: noise

- Many image processing problems can be formulated as inverse problems.
- Most inverse problems are ill-posed, therefore without any prior information, it is almost impossible for solvers to obtain a good solution.

- Current Methods:
  - Classical: **RED**[3]=> no dataset information
  - Neural Network: **UNet**=> no model

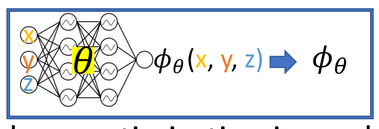
***Idea:*** *We want to meta-learn an good initialization for the classical solver using dataset information.*

## Related Works

### Implicit Representation:

- MLPs with sinusoidal activation functions (**SIREN**) can represent complex natural signals[1].

### Meta-learning – MAML:




- Learn to Learn: an outer loop optimization is used to improve inner loop optimization[2].

[1] Sitzmann, V., Martel, J., Bergman, A., Lindell, D., & Wetzstein, G. (2020). Implicit Neural Representations with Periodic Activation
[2] Chelsea, F. and Pieter A. and Sergey L. (2017) Model-Agnostic Meta-Learning for Fast Adaptation of Deep Networks
[3] Yaniv R. and Michael E. and Peyman M. (2016) The Little Engine that Could: Regularization by Denoising (RED)

**Algorithm 1:** Training Scheme

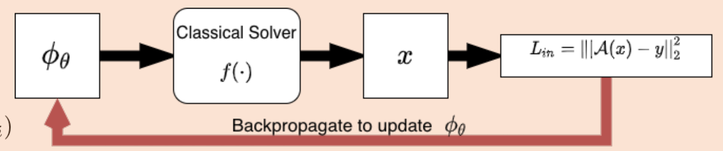
**Require:** Distribution $\mathcal{T}$ over the ground truth signals, forward model $\mathcal{A}(\cdot)$, inner-loop learning rate $\alpha$, number of inner-updates $m$, outer learning rate $\beta$

Initialize meta-parameter $\theta$
**while** *not done* **do**
  sample a batch of signals $x_i \sim \mathcal{T}$
  **for** *all* $x_i$ **do**
    $y_i = \mathcal{A}(x_i) + \epsilon$
    $\theta_i^0 = \theta$
    **for** $k \leftarrow 0$ **to** $m-1$ **do**
      $x = f(\phi_{\theta_i^k})$
      $L_{in}(\theta_i^k, y_i) = \|\mathcal{A}(x) - y_i\|_2^2$
      $\theta_i^{k+1} \leftarrow \theta_i^k - \alpha \nabla_{\theta_i^k} L_{in}(\theta_i^k, y_i)$
    **end**
    $x = f(\phi_{\theta_i^m})$
    $L_{outer} \leftarrow L_{outer} + \|x - x_i\|_2^2$
  **end**
  $\theta \leftarrow \theta - \beta \nabla_\theta L_{outer}$
**end**

## Method

**Idea:** We hope to meta-learn a $\theta$, the parameter of the SIREN, so that the final solution from the proposed inner loop optimization is close to the ground truth signal.

**Inner loop optimization**




* Note: optimize $L_{in}$ without any prior is not sufficient, as the problem is ill-posed. Therefore, we wrap the the optimization procedure in pink box by a meta-learning framework.

**Outer-loop optimization**

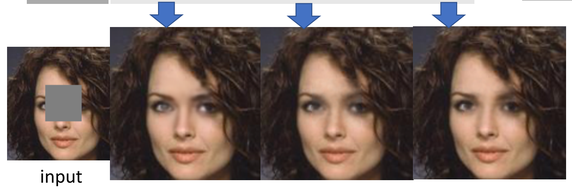
update $\theta$ in the direction that minimize $L_{out} = \|x - x_i\|_2^2$ → the difference between ground truth signal $x_i$ , and the predicted signal from inner loop optimization $x$

## Experimental Results

We perform experiments on two low level image processing tasks with CelebA dataset:

1. Inpainting: Classical Solver: gradient decent
   (we test on different number of inner-loop updates for $\theta$: 3/5 times (3/5-Meta)
2. Deblurring: Classical Solver: RED + pre-trained CNN denoiser
   (MetaRED+CNN: RED updates 3 times, $\theta$ updates 3 times)

| Inpainting | U-Net | 3-Meta | 5-Meta |
|---|---|---|---|
| PSNR | **32.9** | 32.3 | 32.6 |
| SSIM | **0.966** | 0.962 | 0.965 |
| LPIPS | **0.0226** | 0.0251 | 0.0244 |
| MSE | **6.8e-4** | 7.8e-4 | 7.5e-4 |




| Deblurring | MetaRED+CNN | RED + CNN(12) | RED+CNN |
|---|---|---|---|
| PSNR | **29.4** | 27.7 | 29.3 |
| SSIM | **0.918** | 0.898 | 0.913 |
| LPIPS | **0.0504** | 0.119 | 0.0801 |
| MSE | **1.2e-3** | 1.8e-3 | 1.3e-3 |

## Discussion

- We wrap classical solvers with meta-learning to learn a prior across the dataset.
- From deblurring task, our approach gives us: better performance compare to run classical solver with fixed number of iterations; similar performance with RED run till convergence.
- For inpainting task, our method gives meaningful output but does not beat the baseline like U-Net.

- Future Direction:
  - Find a better way to incorporate prior into the inner loop optimization.