Lecture Review and Quizzes (Due: Wednesday, February 19, 1:30pm)
Please review what you have learned in class and then complete the online quiz questions for the following sections:

- Keypoint Detection
- Scale-Space Image Processing

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Homework #6
Released: Monday, February 10
Due: Wednesday, February 19, 1:30pm

1. Robustness of Harris Keypoints to Rotation and Scaling (Total of 10 points)

In this problem, we will evaluate the robustness of the Harris keypoint detector to rotation and scaling. Please download the images hw6_cover_1.jpg and hw6_cover_2.jpg from the handouts webpage. Please answer the following questions for each image.

(a) Apply a Harris keypoint detector and threshold the cornerness response so that about 450-850 Harris keypoints are detected. Please submit a result showing the detected Harris keypoints superimposed on the original image, and report the threshold that you choose. Describe which objects or regions in the image seem to generate large numbers of Harris keypoints. MATLAB functions cornermetric and imregionalmax.

(b) Rotate the image in increments of 15 degrees, from 0 degrees to 360 degrees. For each rotated image, compute Harris keypoints using the same settings that you chose in part (a). Then, compute repeatability using the following procedure.

- Set “number of feature matches” to be 0.
• For each Harris keypoint at \([x, y]\) in the original image:
  o Predict the position \([x_r, y_r]\) where the keypoint should ideally appear in the rotated image.
  o Search for a nearby Harris keypoint which is detected in the rotated image with coordinates \([x_o, y_o]\) satisfying \(|x_o - x_r| \leq 2\) and \(|y_o - y_r| \leq 2\).
  o If such an \([x_o, y_o]\) is found, increment “number of feature matches” by 1.
• Compute repeatability as (number of feature matches) / (number of Harris keypoints in the original image).

Plot repeatability against rotation angle and comment on the Harris keypoint detector’s robustness against rotation.  

(3 points)

(c) Conduct an experiment analogous to part (b), but instead of rotating the image, resize the image by the scaling factors \(m^0, m^1, m^2, \ldots, m^8\), where \(m = 1.2\). Compute Harris keypoints for each resized image. Perform bicubic interpolation with MATLAB function \texttt{imresize}. By comparing the Harris keypoints of the original image and the corresponding Harris keypoints of the resized image, compute and plot repeatability against scaling factor (log scale may be most appropriate), and comment on the Harris keypoint detector’s robustness against scale changes.  

(3 points)

Note: Please attach relevant MATLAB code.
2. Robustness of SIFT Keypoints to Rotation and Scaling (Total of 10 points)

In this problem, we will evaluate the robustness of the SIFT keypoint detector to rotation and scaling. The VLFeat library (http://www.vlfeat.org) contains an implementation of the full SIFT algorithm (see: http://www.vlfeat.org/overview/sift.html), including both the detector and the descriptor. We will just use the detector for this problem.

You can download the VLFeat library from this link:
http://www.vlfeat.org/download.html

To access VLFeat from within MATLAB, please use the instructions here:
http://www.vlfeat.org/install-matlab.html

For the images hw6_cover_1.jpg and hw6_cover_2.jpg from Problem 1, please answer the following questions.

(a) Apply a SIFT keypoint detector and adjust the peak and edge thresholds so that about 450-850 SIFT keypoints are detected. Please submit a result showing the detected SIFT keypoints superimposed on the original image (function: vl_plotframe), and report the thresholds you choose. Describe which objects or regions in the image seem to generate large numbers of SIFT keypoints.

(4 points)

(b) Using the same procedure defined in Problem 1, plot repeatability versus rotation angle (in increments of 15 degrees, from 0 degrees to 360 degrees). Comment on the SIFT keypoint detector's robustness against rotation and compare against the corresponding result for the Harris keypoint detector.

(3 points)

(c) Using the same procedure defined in Problem 1, plot repeatability versus scaling factor (with the scaling factors m^0, m^1, m^2, ..., m^8, where m = 1.2). Comment on the SIFT keypoint detector's robustness against scale changes and compare against the corresponding result for the Harris keypoint detector.

(3 points)

Note: Please attach relevant MATLAB code, excluding the code already provided in the VLFeat library.
3. Scale Selection for Determinant-of-Hessian (Total of 8 points)

Consider a continuous-space version of the determinant-of-Hessian (DoH) keypoint detector. For an input image \( f(x, y) \), the DoH response \( \det H'(x, y) \) at scale \( t \) is defined as follows:

\[
\det H'(x, y) = f'_{xx}(x, y)f'_{yy}(x, y) - \left(f'_{xy}(x, y)\right)^2
\]

\[
f'_{xx}(x, y) = \frac{\partial^2}{\partial x^2} g'(x, y) \ast f(x, y)
\]

\[
f'_{yy}(x, y) = \frac{\partial^2}{\partial y^2} g'(x, y) \ast f(x, y)
\]

\[
f'_{xy}(x, y) = \frac{\partial^2}{\partial y \partial x} g'(x, y) \ast f(x, y)
\]

\[
g'(x, y) = \frac{1}{2\pi t} \exp \left(-\frac{x^2 + y^2}{2t}\right)
\]

(a) Assume the input image is a Gaussian blob of scale \( t_0 > 0 \) : \( f(x, y) = 2\pi t_0 \cdot g^b(x, y) \). From the above definitions, derive and report an expression for the following objective functions:

- \( H_A(t) = \det H'(0,0) \)
- \( H_B(t) = t \det H'(0,0) \)
- \( H_C(t) = t^2 \det H'(0,0) \)

(4 points)

(b) Derive and report the scale values \( t_A^* \geq 0 \), \( t_B^* \geq 0 \), and \( t_C^* \geq 0 \) that maximize \( H_A(t) \), \( H_B(t) \), and \( H_C(t) \), respectively. Also report the maximal values \( H_A\left(t_A^*\right) \), \( H_B\left(t_B^*\right) \), and \( H_C\left(t_C^*\right) \). Please interpret the results.

(4 points)