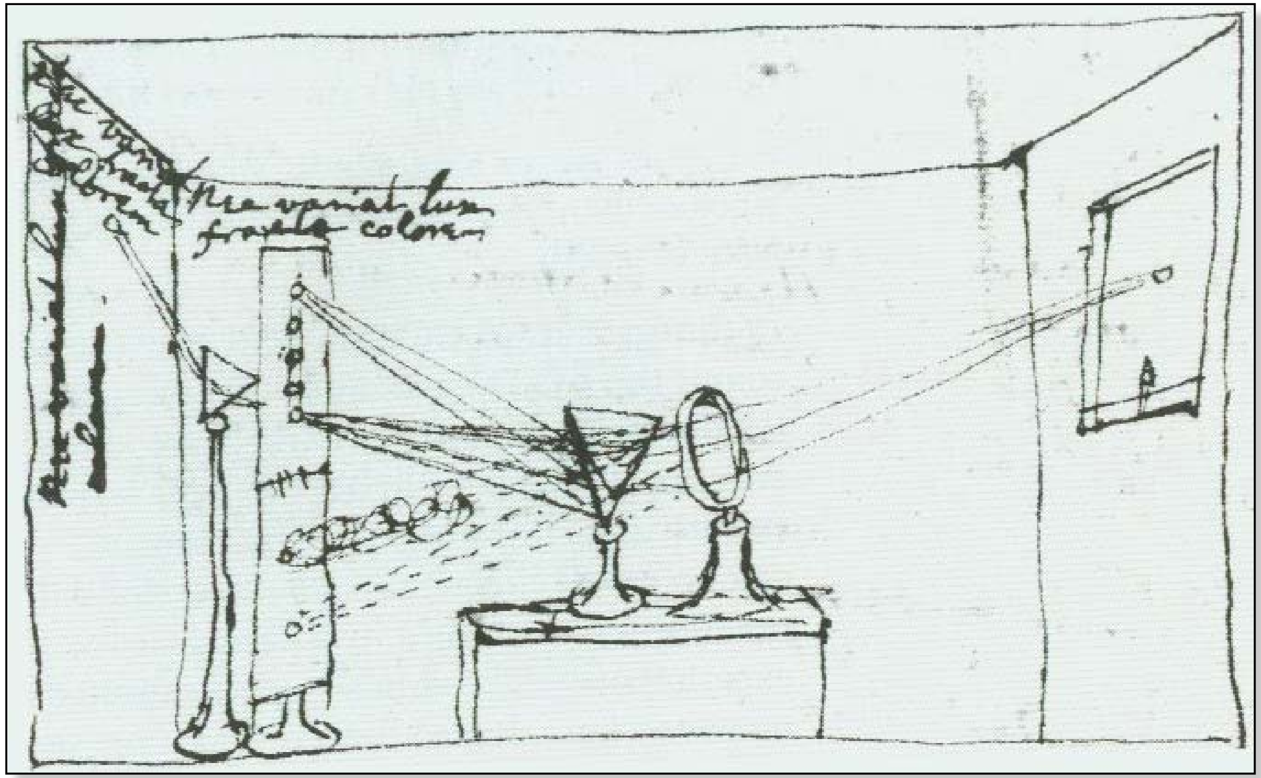
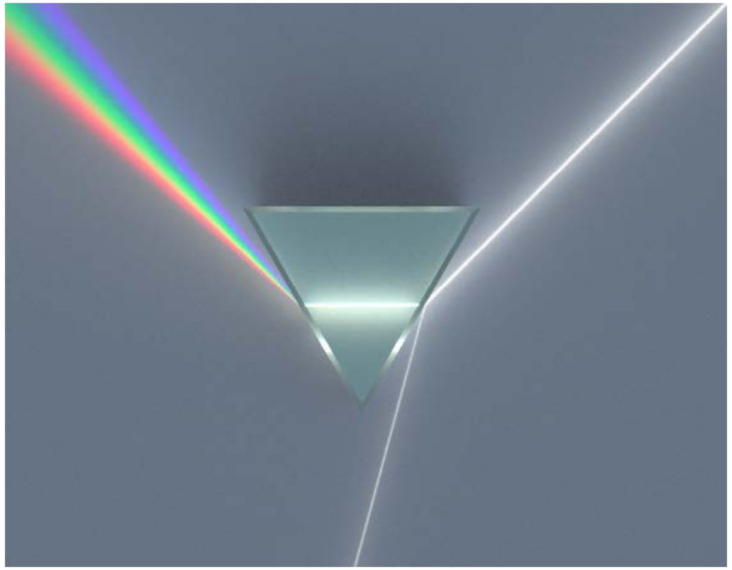
Introduction to color science

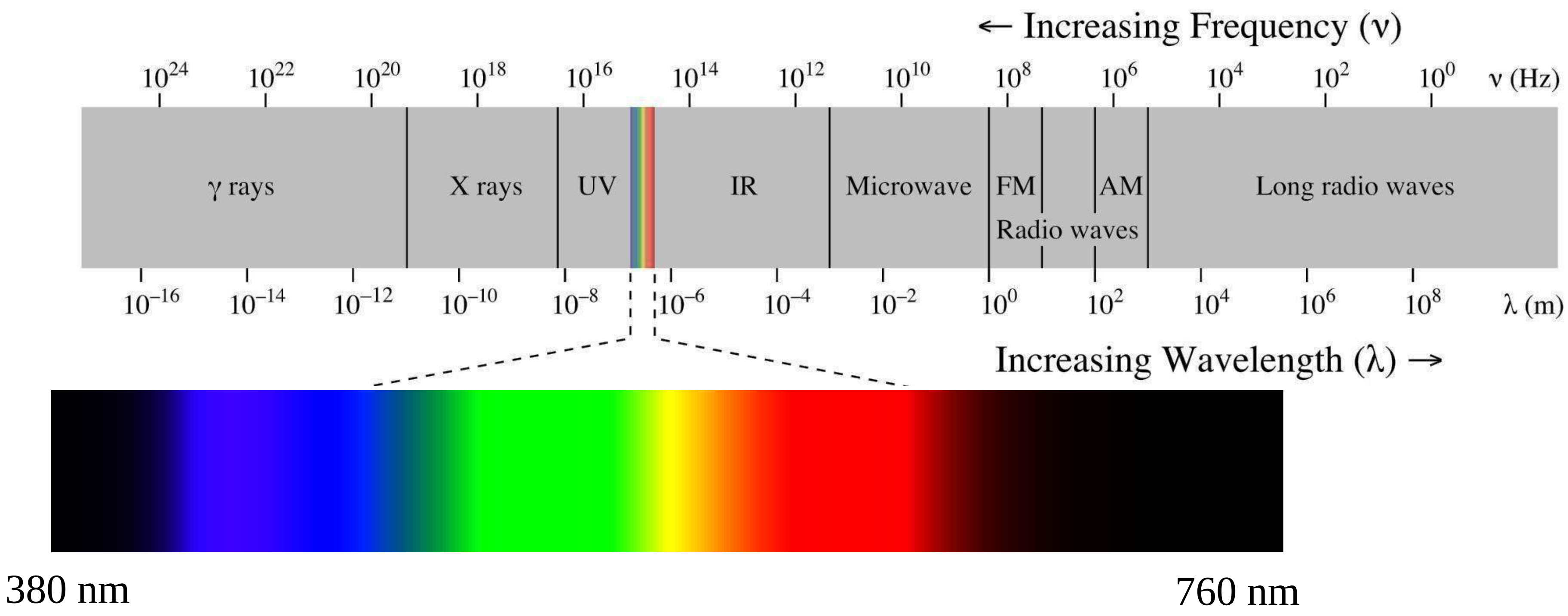
- Trichromacy
- Spectral matching functions
- CIE XYZ color system
- xy-chromaticity diagram
- Color gamut
- Color temperature
- Color balancing algorithms

Newton’s Prism Experiment - 1666

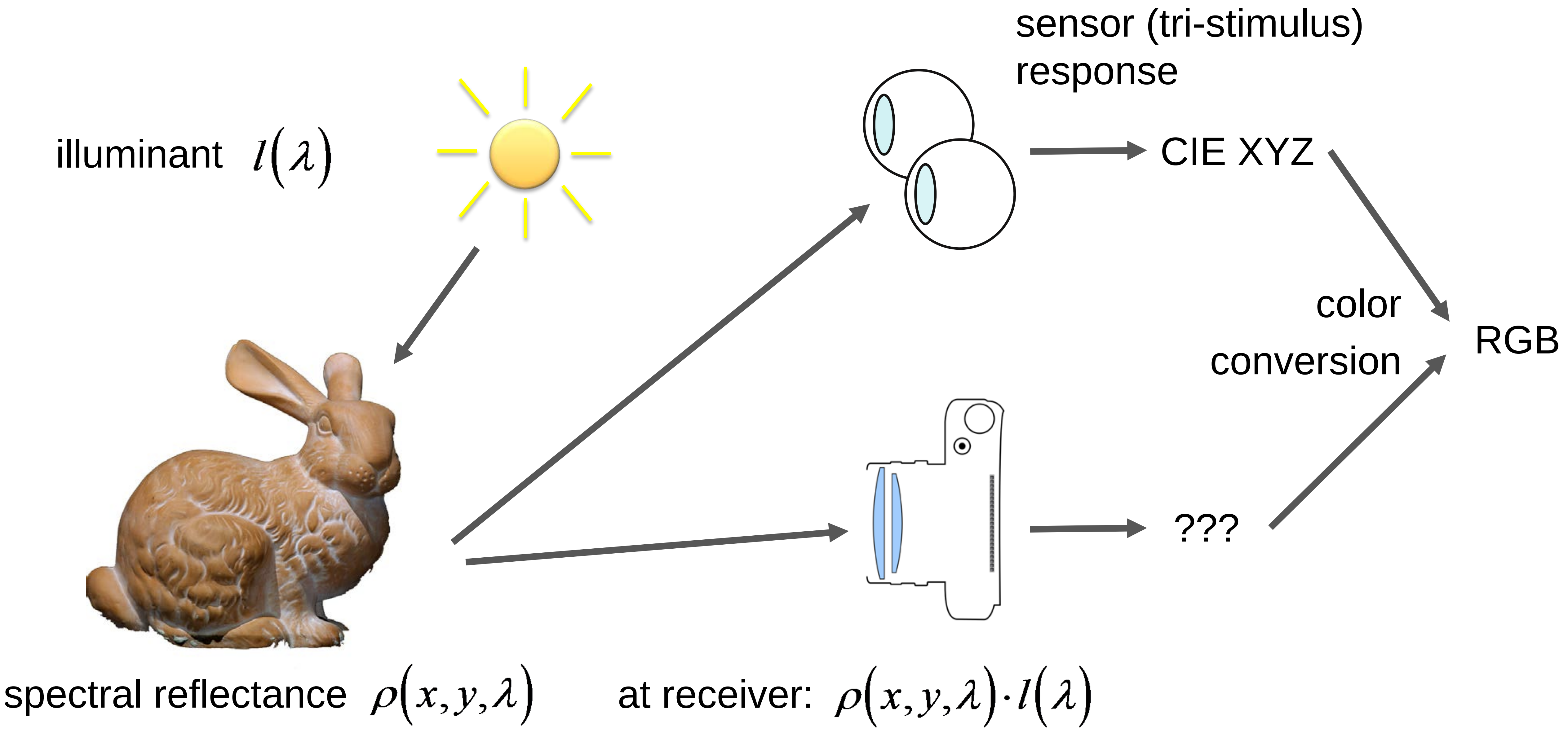




Color: visible range of the electromagnetic spectrum



Radiometry overview



Radiometric Quantities

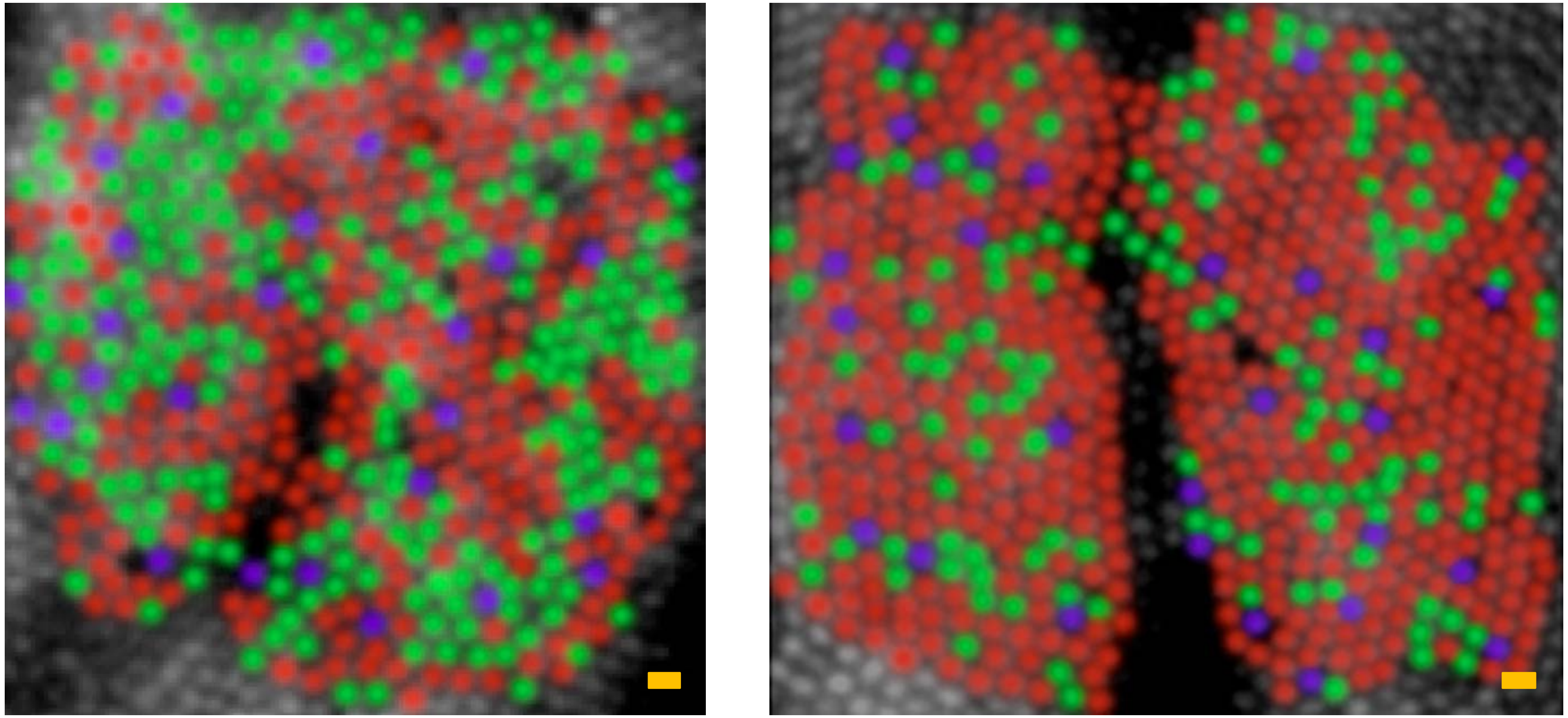
Quantity	Symbol	Unit name	Unit symbol	Dimension	Notes
Radiance	$L_{e,\Omega}$ [nb 5]	watt per steradian per square metre	$W \cdot sr^{-1} \cdot m^{-2}$	$\mathbf{M} \cdot \mathbf{T}^{-3}$	Radiant flux emitted, reflected, transmitted or received by a *surface*, per unit solid angle per unit projected area. This is a *directional* quantity. This is sometimes also confusingly called "intensity".
Spectral radiance	$L_{e,\Omega,\nu}$ [nb 3] or $L_{e,\Omega,\lambda}$ [nb 4]	watt per steradian per square metre per hertz *or* watt per steradian per square metre, per metre	$W \cdot sr^{-1} \cdot m^{-2} \cdot Hz^{-1}$ *or* $W \cdot sr^{-1} \cdot m^{-3}$	$\mathbf{M} \cdot \mathbf{T}^{-2}$ *or* $\mathbf{M} \cdot \mathbf{L}^{-1} \cdot \mathbf{T}^{-3}$	Radiance of a *surface* per unit frequency or wavelength. The latter is commonly measured in $W \cdot sr^{-1} \cdot m^{-2} \cdot nm^{-1}$. This is a *directional* quantity. This is sometimes also confusingly called "spectral intensity".
Irradiance	E_e [nb 2]	watt per square metre	W/m^2	$\mathbf{M} \cdot \mathbf{T}^{-3}$	Radiant flux *received* by a *surface* per unit area. This is sometimes also confusingly called "intensity".

Quantity Name	Quantity Symbol[nb 1]	Unit Name	Unit Symbol	Dimension Symbol	Notes
Radiant energy					
Radiant energy dens					
Radiant flux					
Spectral flux					
Radiant intensity					
Spectral intensity					
Radiance					
Spectral radiance					
Irradiance					
Spectral irradiance					
Radiosity					
Spectral radiosity					
Radiant exitance					
Spectral exitance	or $M_{e,\lambda}$ [nb 4]	*or* watt per square metre, per metre	*or* W/m^3	*or* $\mathbf{M} \cdot \mathbf{L}^{-1} \cdot \mathbf{T}^{-3}$	"Spectral emittance" is an old term for this quantity. This is sometimes also confusingly called "spectral intensity".
Radiant exposure	H_e	joule per square metre	J/m^2	$\mathbf{M} \cdot \mathbf{T}^{-2}$	Radiant energy received by a *surface* per unit area, or equivalently irradiance of a *surface* integrated over time of irradiation. This is sometimes also called "radiant fluence".
Spectral exposure	$H_{e,\nu}$ [nb 3] or $H_{e,\lambda}$ [nb 4]	joule per square metre per hertz *or* joule per square metre, per metre	$J \cdot m^{-2} \cdot Hz^{-1}$ *or* J/m^3	$\mathbf{M} \cdot \mathbf{T}^{-1}$ *or* $\mathbf{M} \cdot \mathbf{L}^{-1} \cdot \mathbf{T}^{-2}$	Radiant exposure of a *surface* per unit frequency or wavelength. The latter is commonly measured in $J \cdot m^{-2} \cdot nm^{-1}$. This is sometimes also called "spectral fluence".

Photometric Quantities

Quantity		Unit		Dimension	Notes
Name	Symbol[nb 1]	Name	Symbol	Symbol	
Luminous energy	Q_v [nb 2]	lumen second	lm⋅s	**T⋅J** [nb 3]	Units are sometimes called *talbots*.
Luminous flux / Luminous power	Φ_v [nb 2]	lumen (= cd⋅sr)	lm	**J** [nb 3]	Luminous energy per unit time.
Luminous intensity	I_v	candela (= lm/sr)	cd	**J** [nb 3]	Luminous power per unit solid angle.
Luminance	L_v	candela per square metre	cd/m^2	$\mathbf{L}^{-2}\cdot\mathbf{J}$	Luminous power per unit solid angle per unit *projected* source area. Units are sometimes called *nits*.
Illuminance	E_v	lux (= lm/m^2)	lx	$\mathbf{L}^{-2}\cdot\mathbf{J}$	Luminous power *incident* on a surface.
Luminous exitance / Luminous emittance	M_v	lux	lx	$\mathbf{L}^{-2}\cdot\mathbf{J}$	Luminous power *emitted* from a surface.
Luminous exposure	H_v	lux second	lx⋅s	$\mathbf{L}^{-2}\cdot\mathbf{T}\cdot\mathbf{J}$	
Luminous energy density	ω_v	lumen second per cubic metre	lm⋅s⋅m^{-3}	$\mathbf{L}^{-3}\cdot\mathbf{T}\cdot\mathbf{J}$	
Luminous efficacy	η [nb 2]	lumen per watt	lm/W	$\mathbf{M}^{-1}\cdot\mathbf{L}^{-2}\cdot\mathbf{T}^{3}\cdot\mathbf{J}$	Ratio of luminous flux to radiant flux.
Luminous efficiency / Luminous coefficient	V			1	

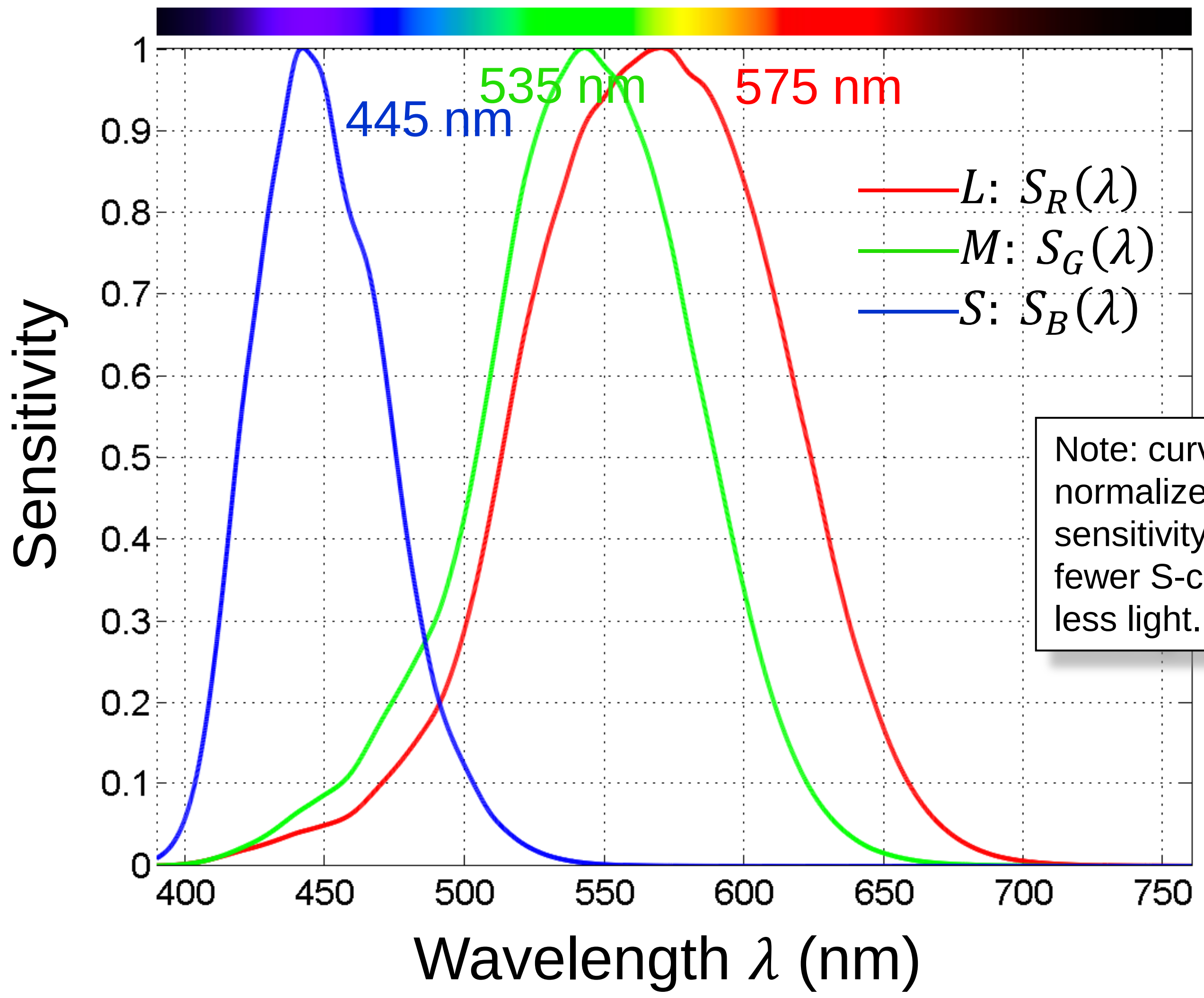
Human retina



[Roorda, Williams, 1999]

Pseudo-color image of nasal retina,
1 degree eccentricity, in two male subjects, scale bar 5 micron

Absorption of light in the cones of the human retina



Note: curves are normalized. **Much** lower sensitivity to Blue, since fewer S-cones absorb less light.

Three-receptor model of color perception

$C(\lambda)$

Spectral energy distribution of incident light

$$\int_{\lambda} S_R(\lambda) C(\lambda) d\lambda \rightarrow \alpha_R$$

$$\int_{\lambda} S_G(\lambda) C(\lambda) d\lambda \rightarrow \alpha_G$$

$$\int_{\lambda} S_B(\lambda) C(\lambda) d\lambda \rightarrow \alpha_B$$

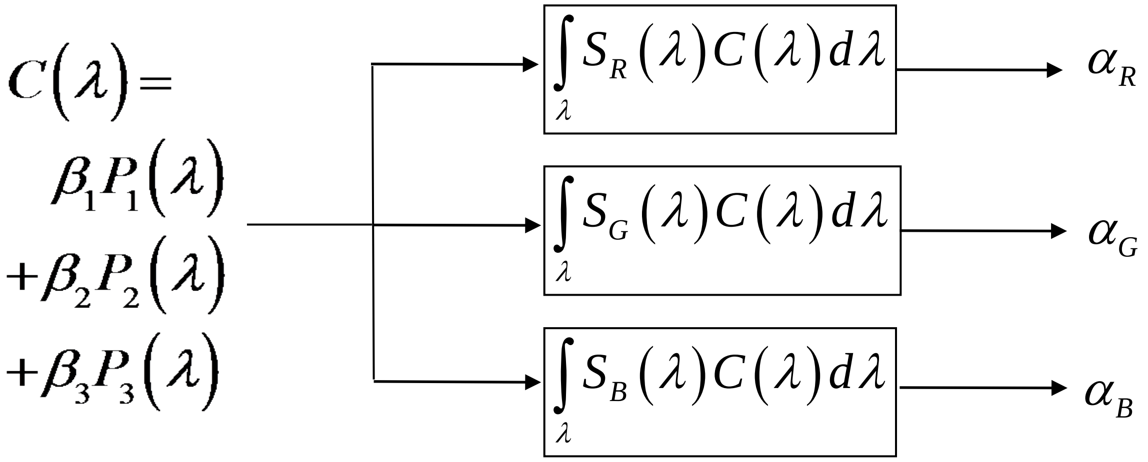
Effective cone stimulation (“tristimulus values”)

[T. Young, 1802] [J.C. Maxwell, 1890]

- Different spectra can map into the same tristimulus values and hence look identical (“metamers”)
- Three numbers suffice to represent any color – Grassmann’s law

Color matching

- Suppose 3 primary light sources with spectra $P_k(\lambda)$, $k = 1,2,3$
- Intensity of each light source can be adjusted by factor β_k
- How to choose β_k, $k = 1,2,3$, such that desired tristimulus values (α_R, α_G, α_B) result ?



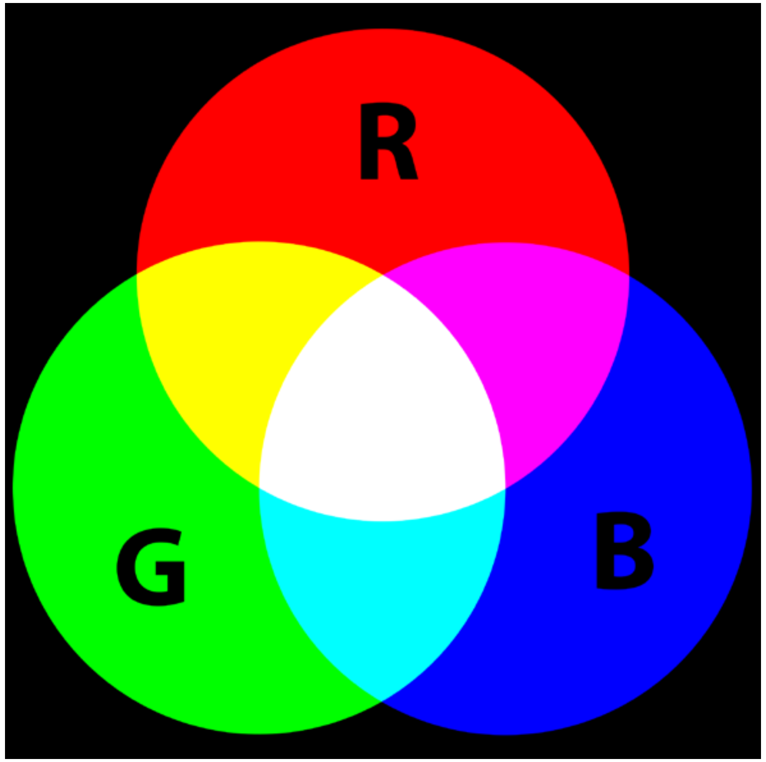
$$\alpha_i = \int_\lambda S_i(\lambda)\left[\beta_1 P_1(\lambda) + \beta_2 P_2(\lambda) + \beta_3 P_3(\lambda)\right] d\lambda$$

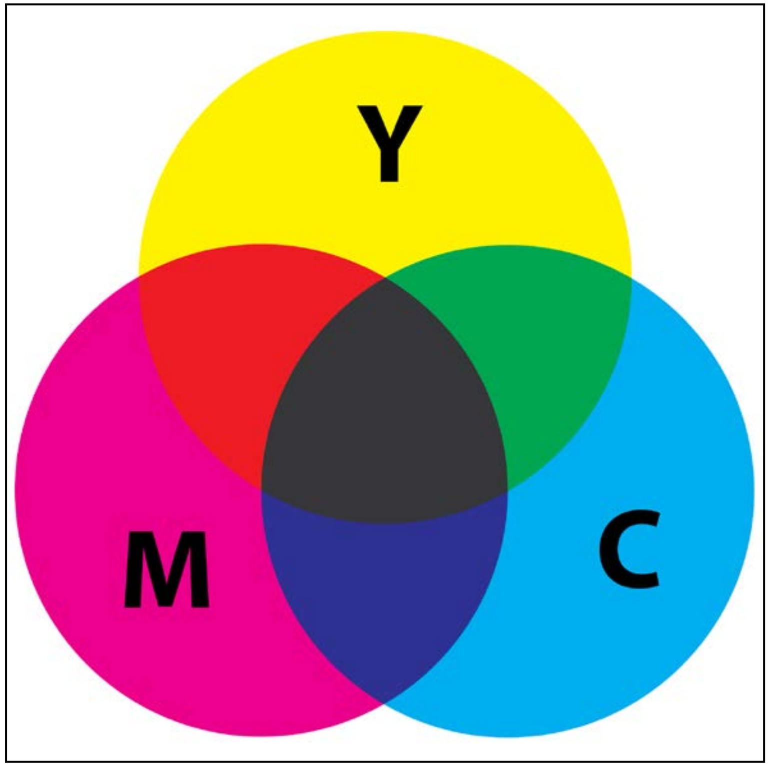
$$= \beta_1 \cdot K_{i,1} + \beta_2 \cdot K_{i,2} + \beta_3 \cdot K_{i,3}$$

with $K_{i,j} = \int_\lambda S_i(\lambda) P_j(\lambda) d\lambda$

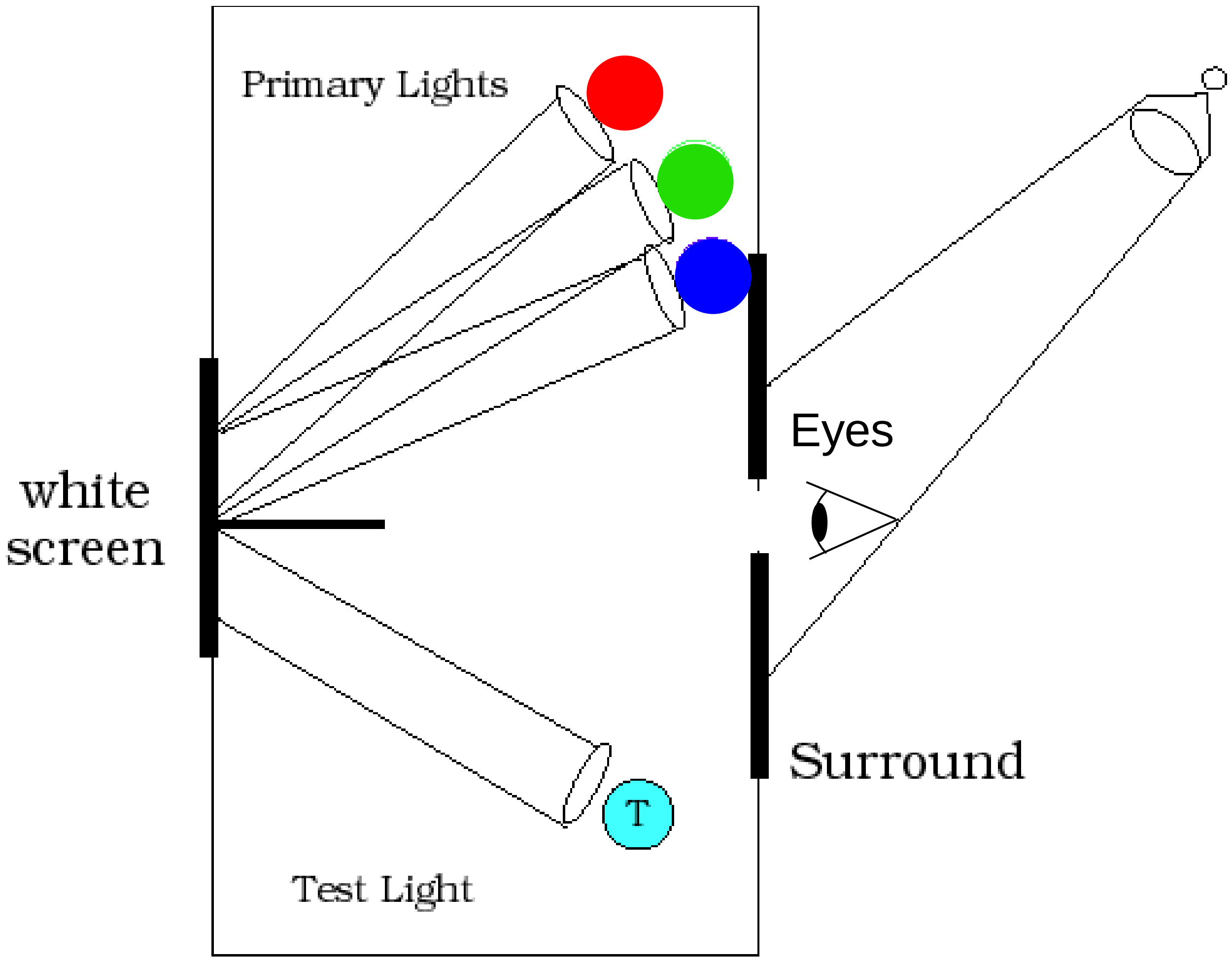
Color matching is linear!

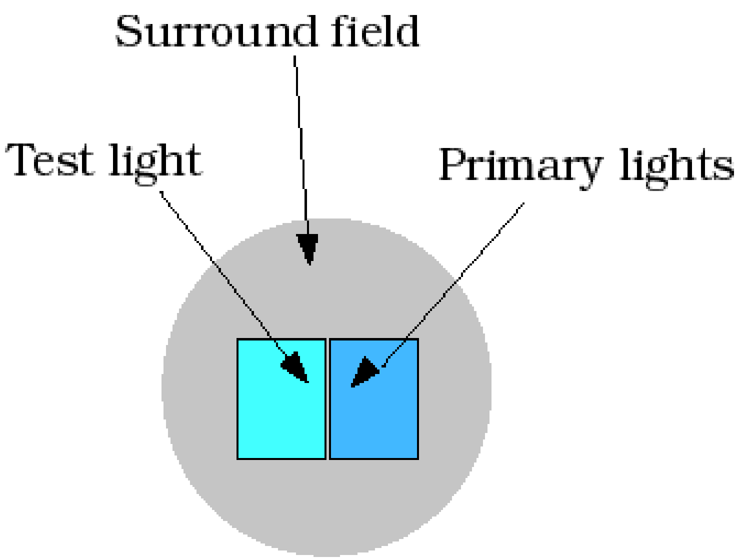
Additive vs. subtractive color mixing





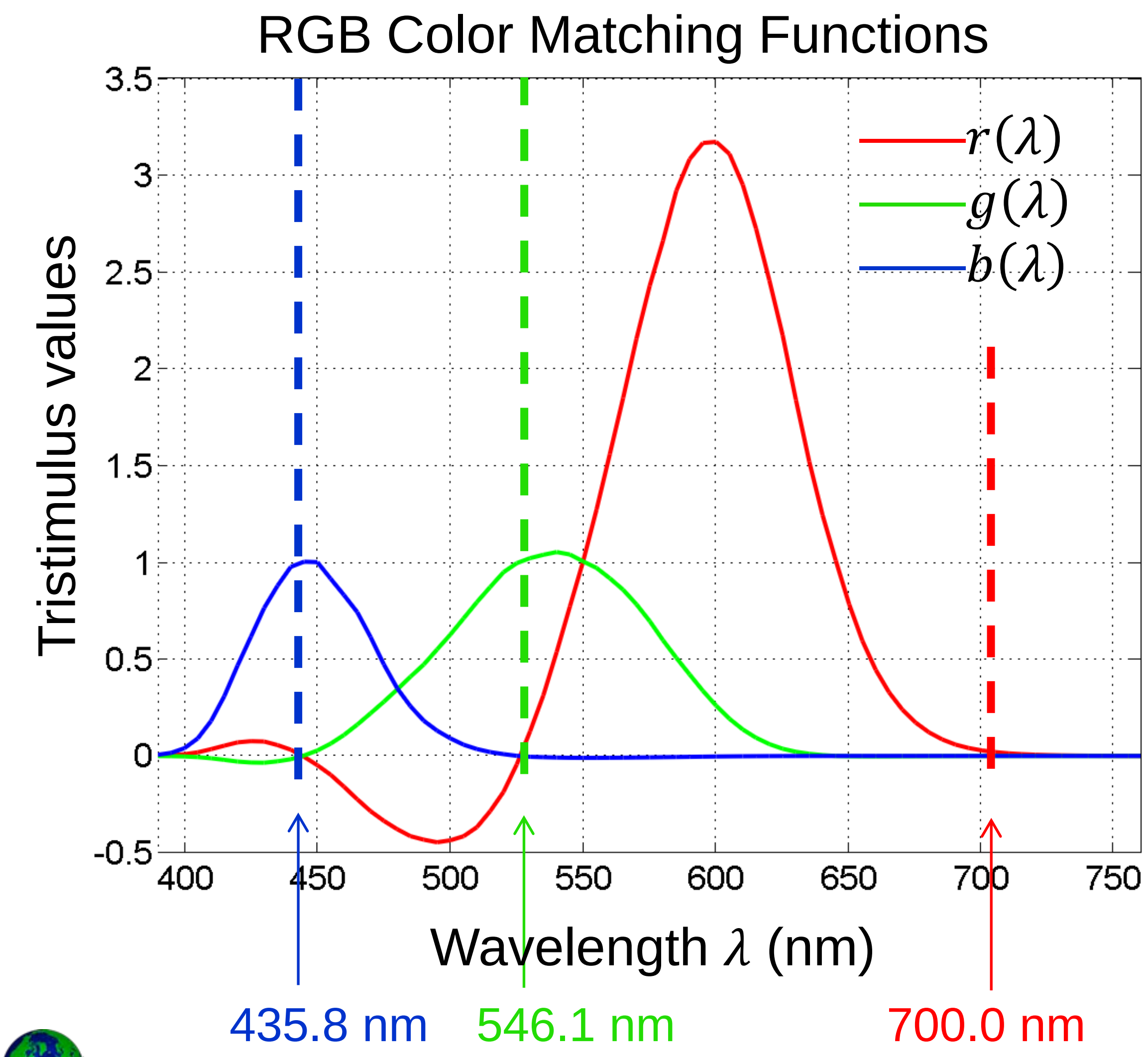
Color matching experiment





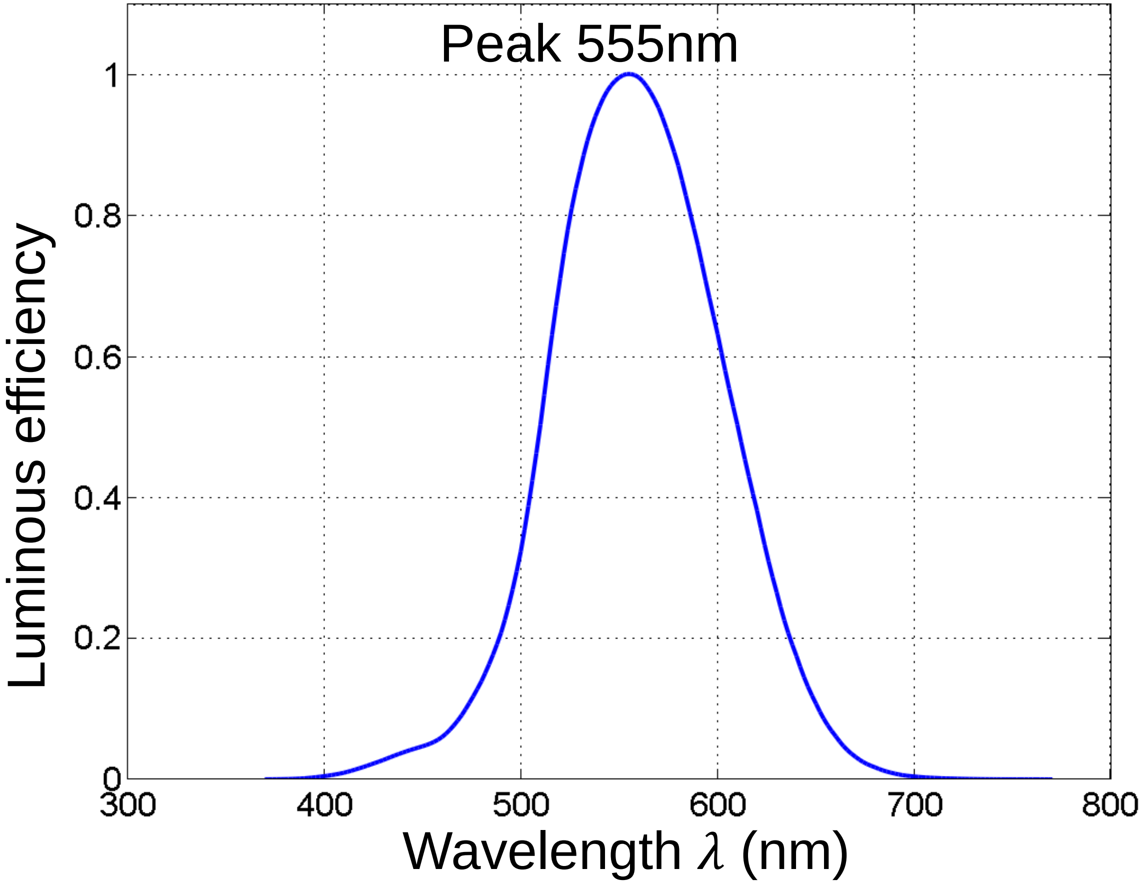
Courtesy B. Wandell, from [Foundations of Vision, 1996]

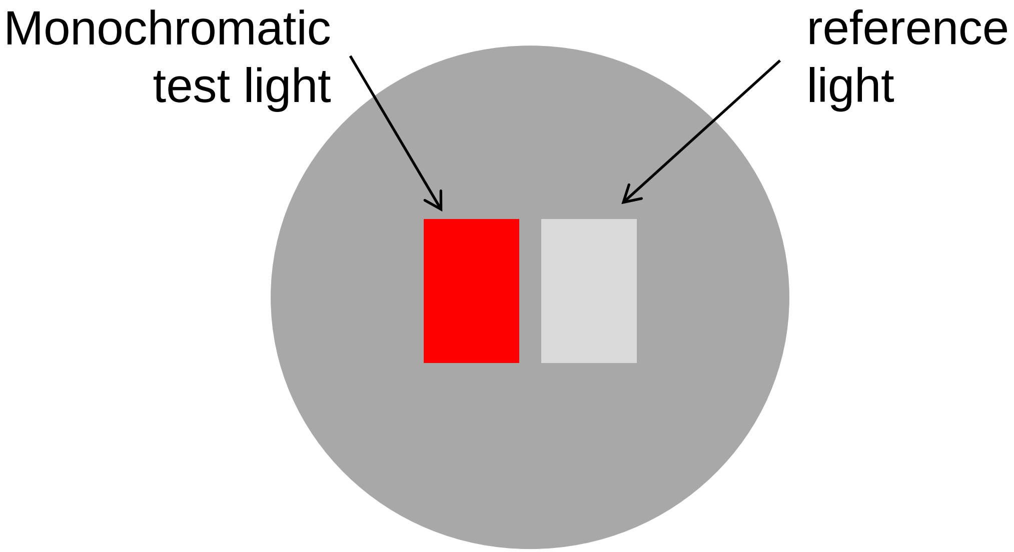
Spectral matching functions



- Color matching experiment: Monochromatic test light and monochromatic primary lights
- Spectral RGB primaries (scaled, such that R=G=B matches spectrally flat white)
- “Negative intensity”: color is added to test color
- Standard human observer: CIE (Commision Internationale de L’Eclairage), 1931.

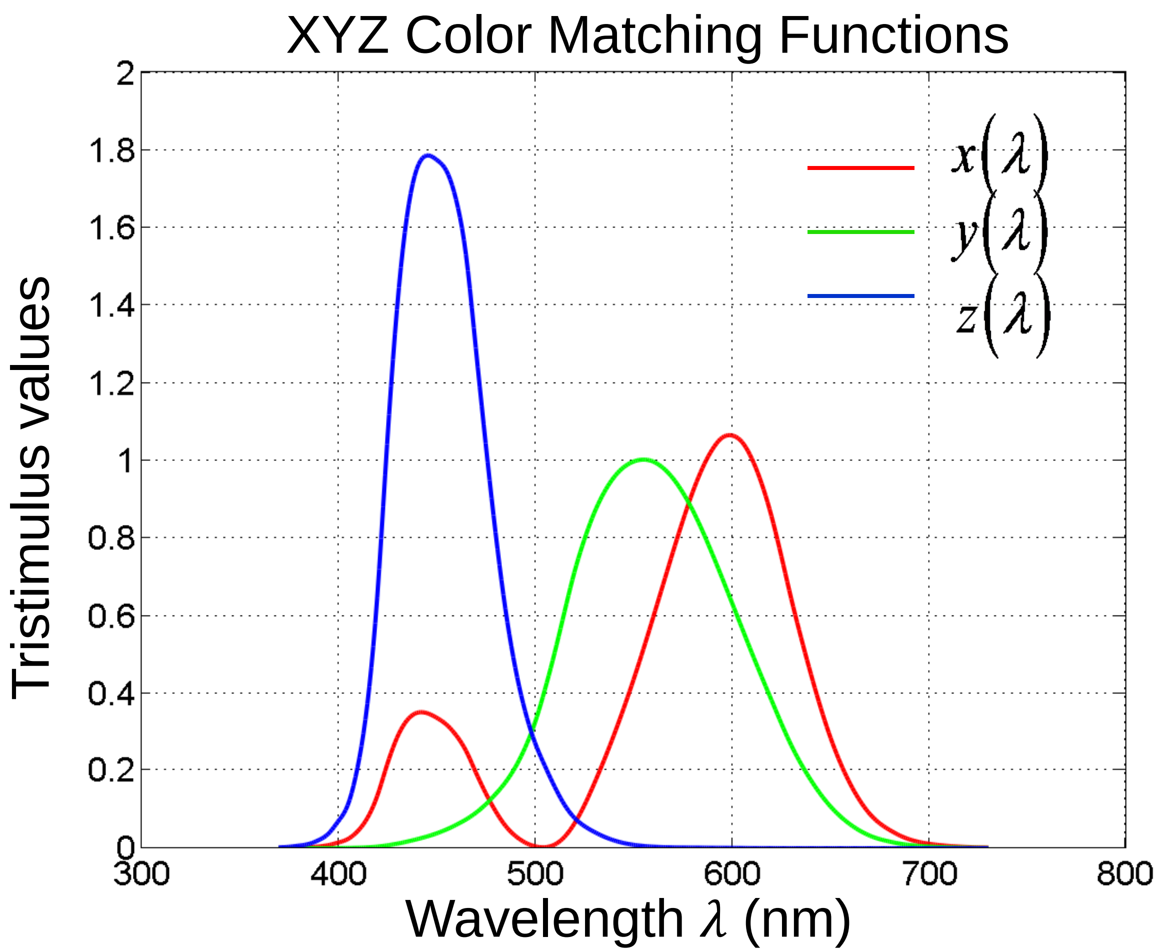
Luminosity function





- Experiment:
  Match the brightness of a white reference light and a monochromatic test light of wavelength λ
- Links photometric to radiometric quantities

CIE 1931 XYZ color system



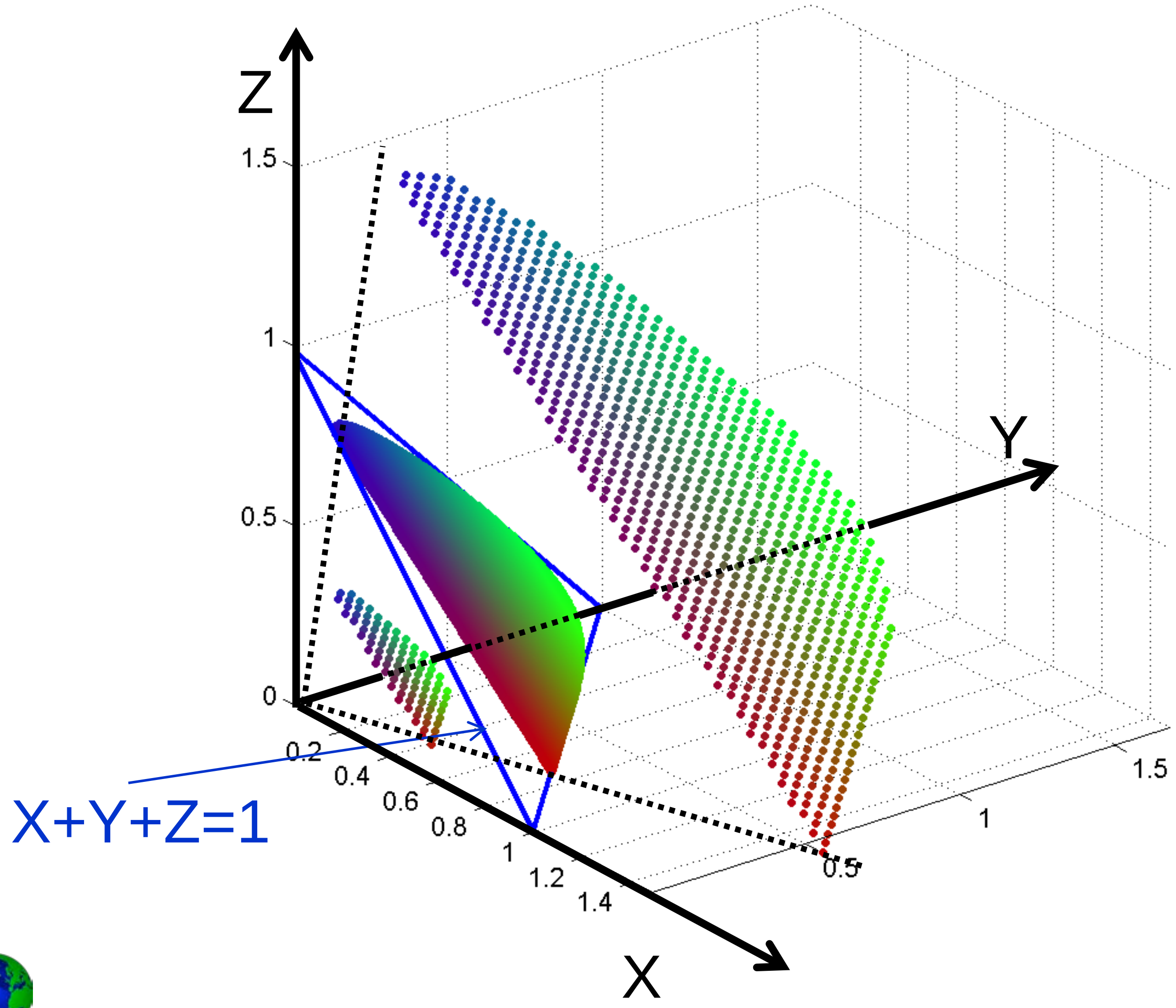
Properties:

- All positive spectral matching functions

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} .490 & .310 & .200 \\ .177 & .813 & .011 \\ .000 & .010 & .990 \end{pmatrix} \begin{pmatrix} R_\lambda \\ G_\lambda \\ B_\lambda \end{pmatrix}$$

- Y corresponds to luminance
- Equal energy white: X=Y=Z
- Virtual primaries

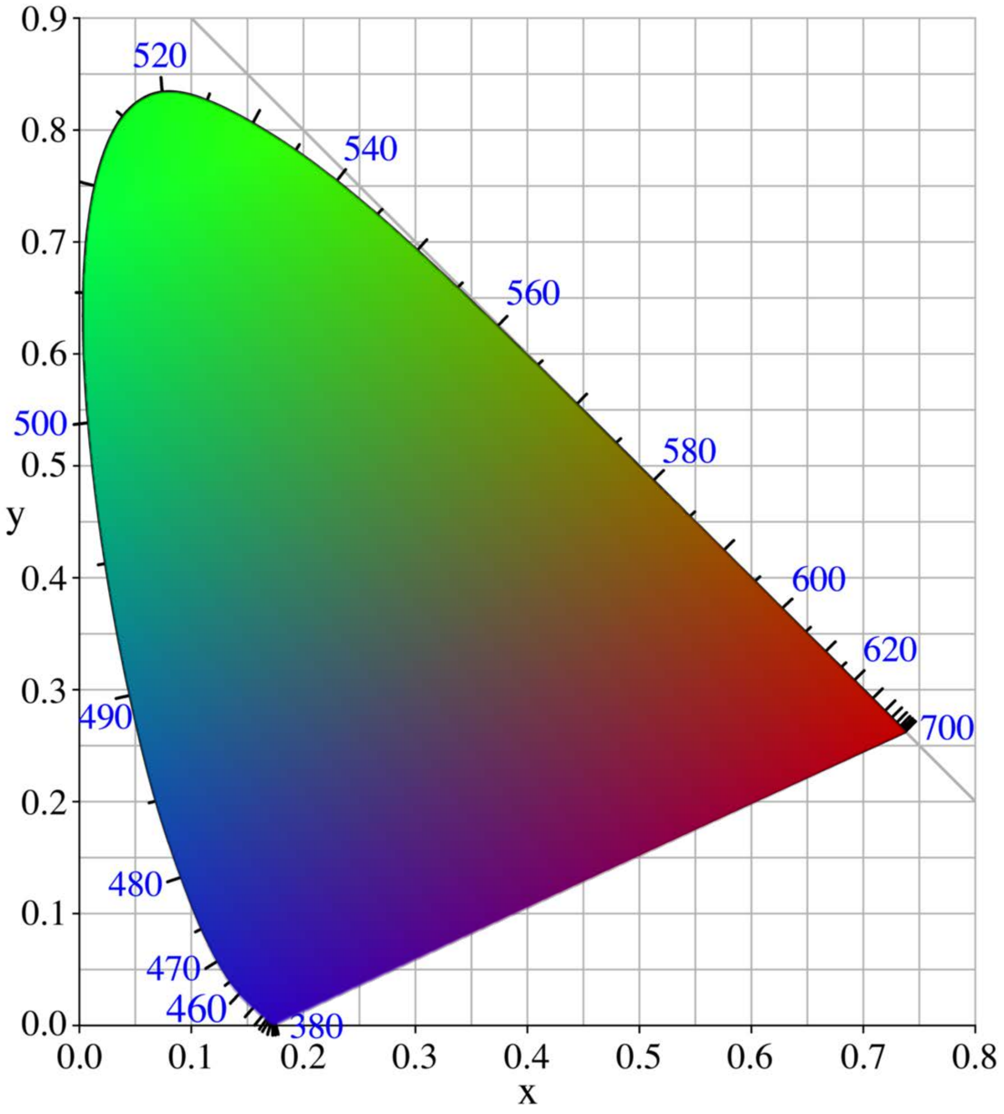
Color gamut and chromaticity



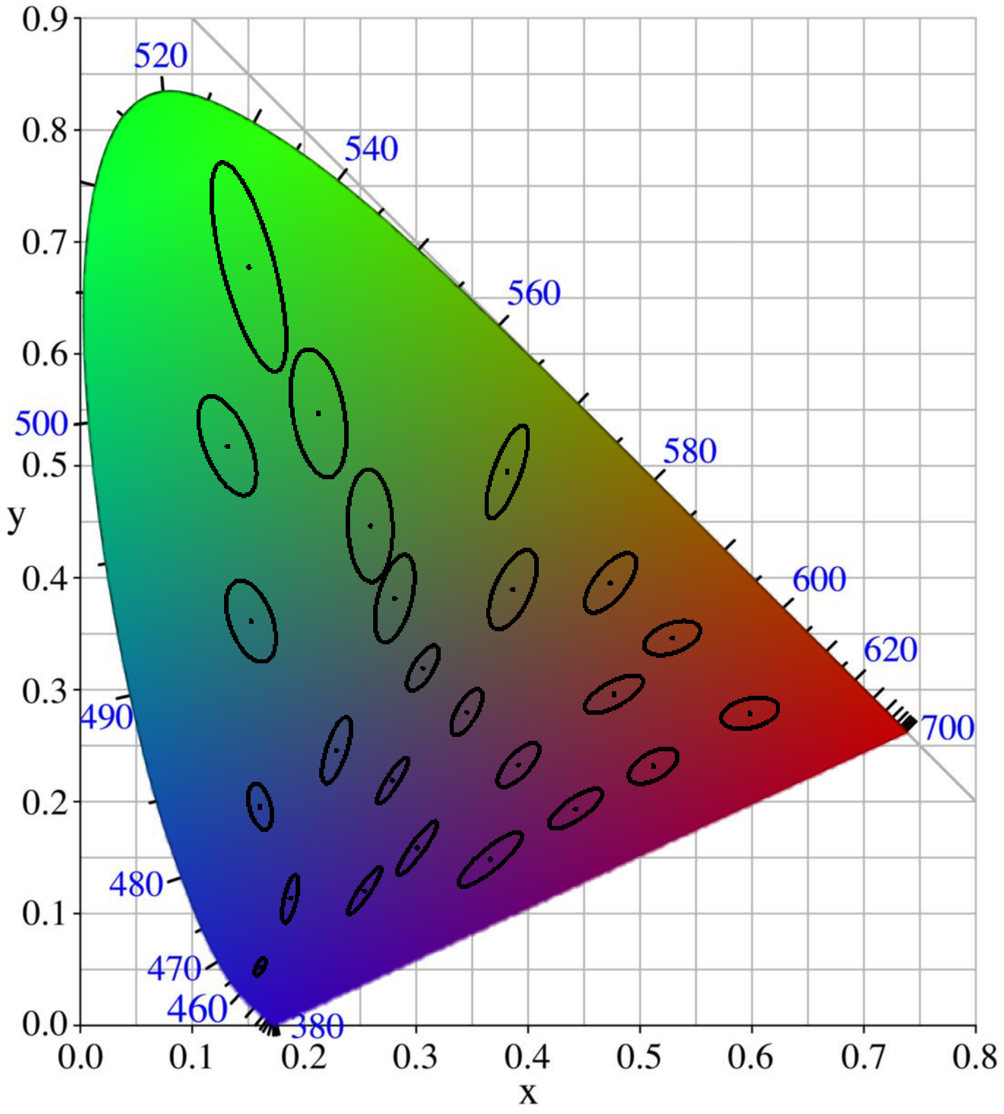
$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

CIE chromaticity diagram



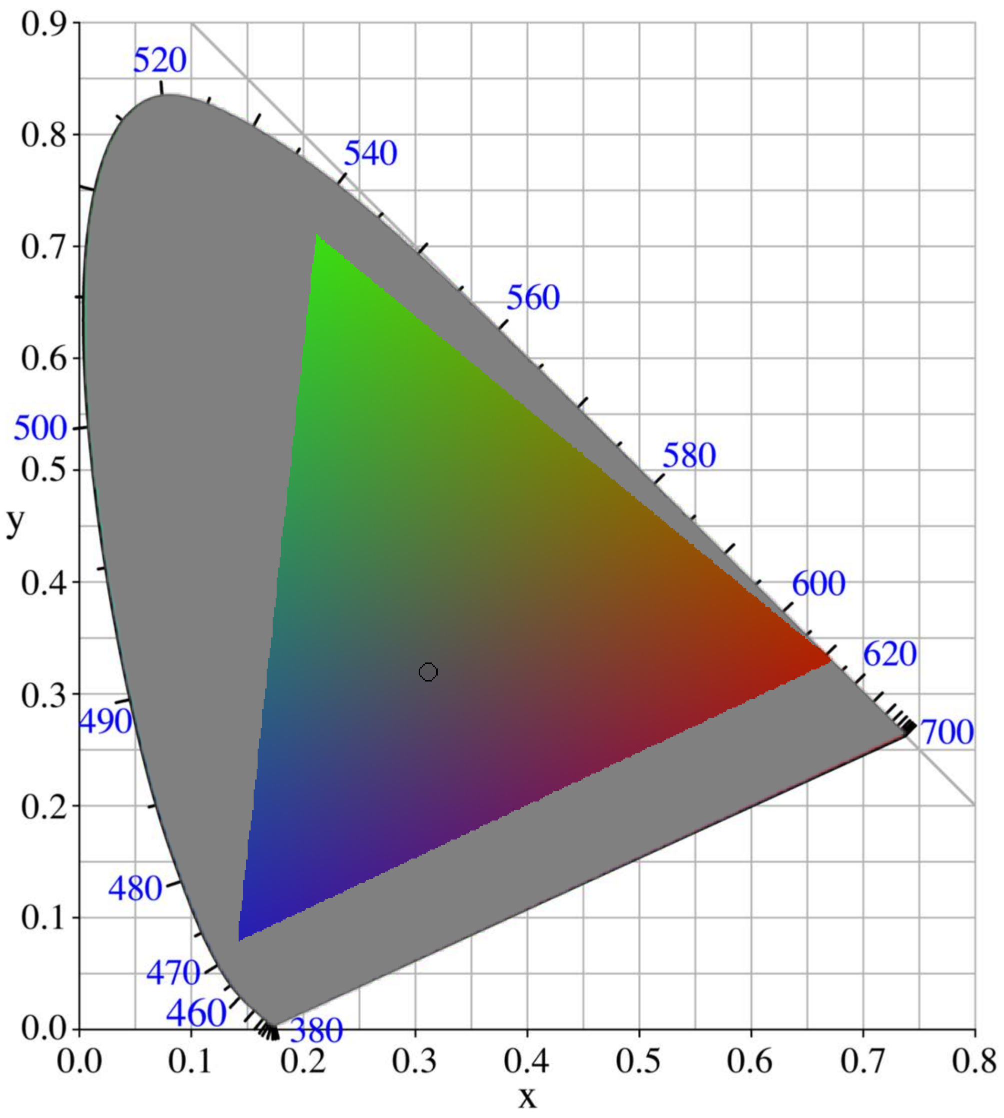
Perceptual non-uniformity of xy chromaticity



Just noticeable chromaticity differences (10X enlarged)

[MacAdam, 1942]

Color gamut

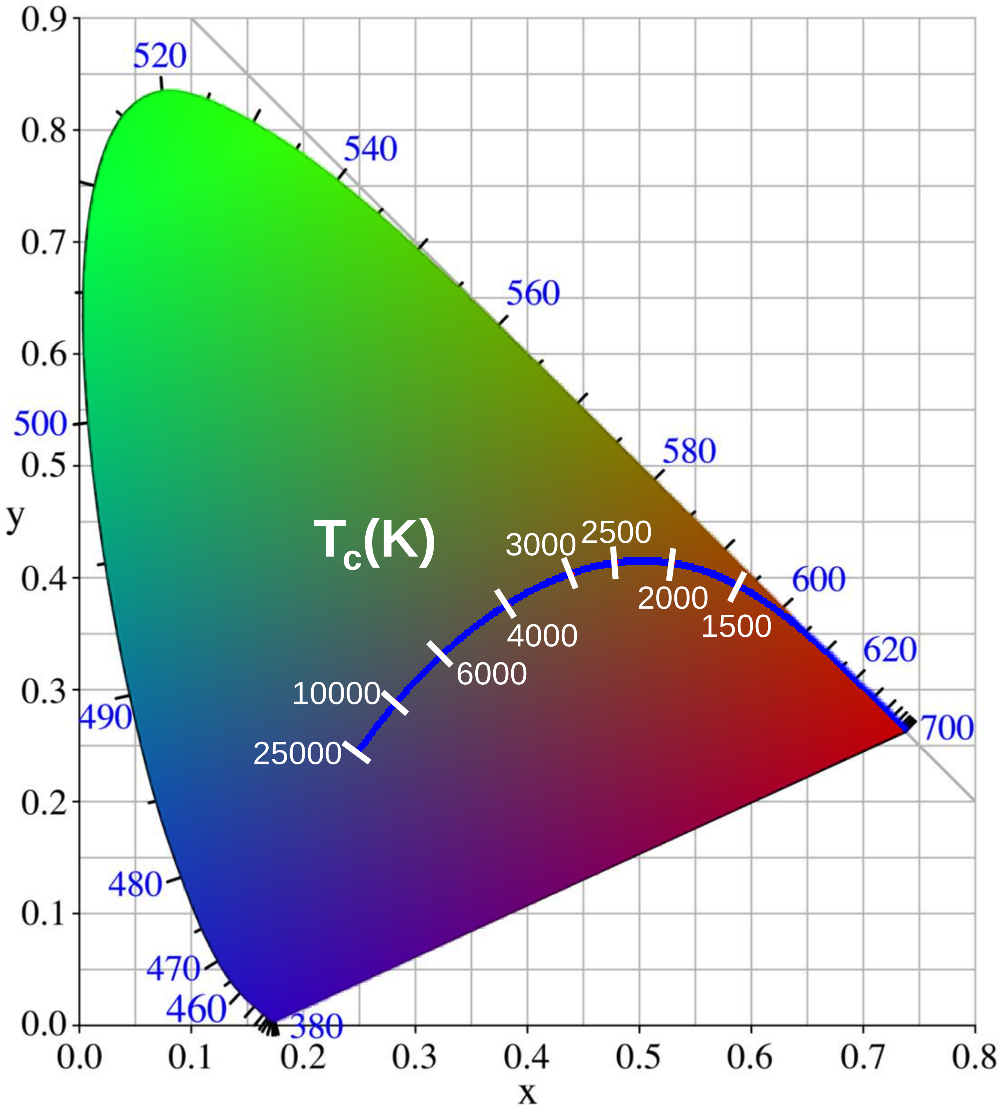


NTSC phosphors

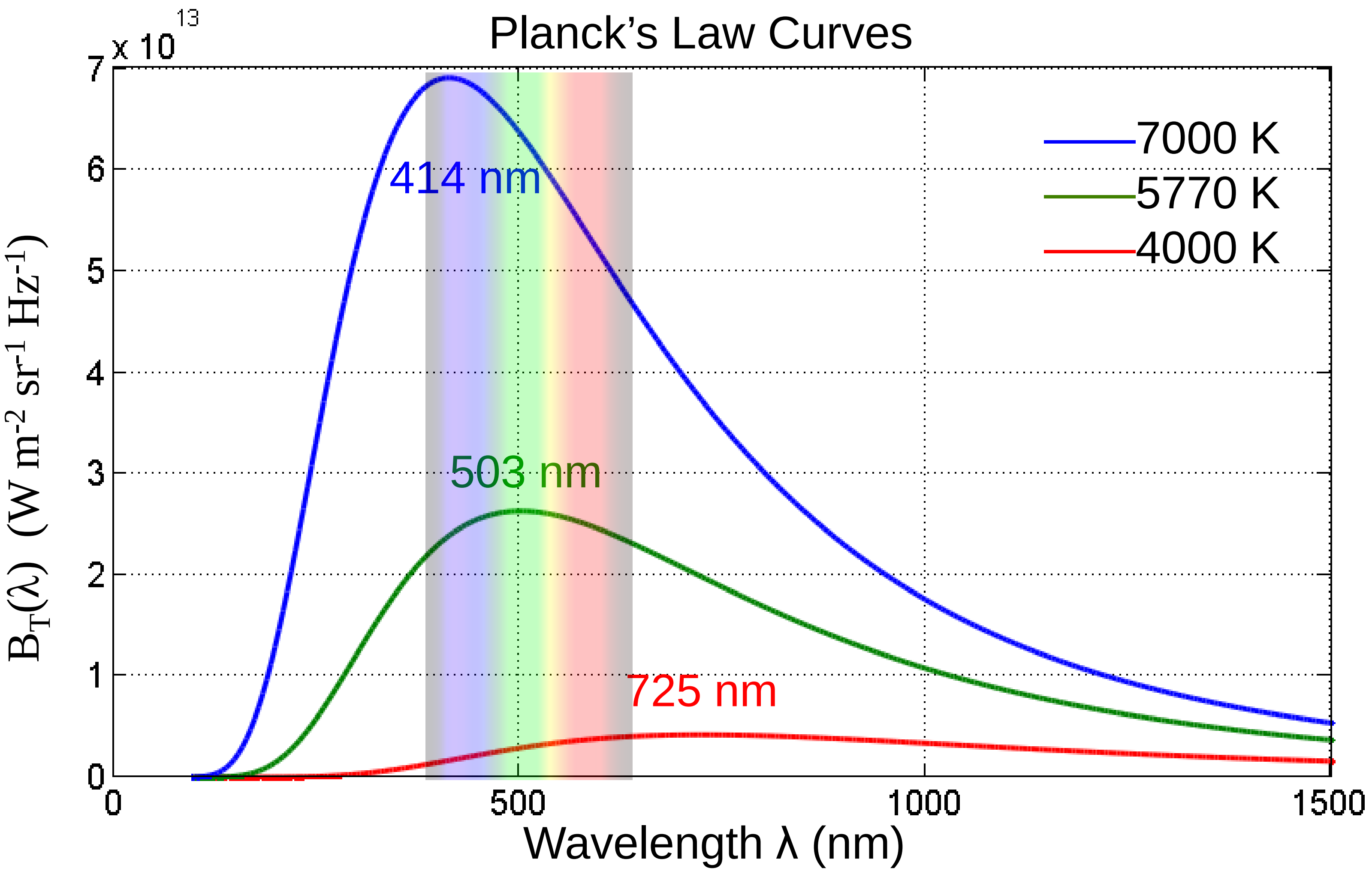
R: x=0.67, y=0.33
G: x=0.21, y=0.71
B: x=0.14, y=0.08

Reference white:
x=0.31, y=0.32
Illuminant C

White at different color temperatures



Blackbody radiation



Planck's Law, 1900

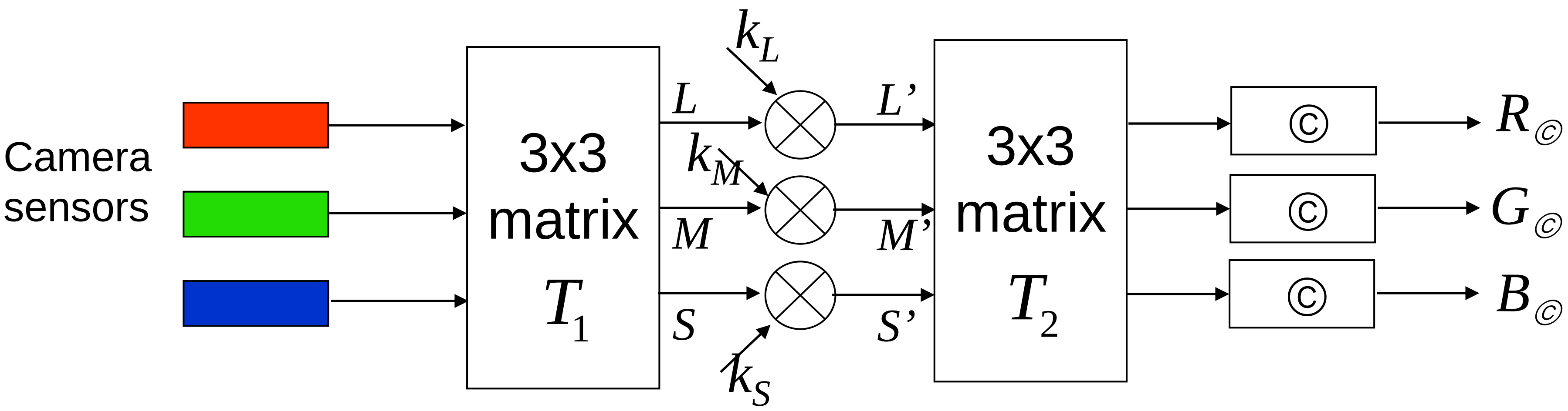
$$B_T(\lambda) = \frac{2hc^2 / \lambda^5}{e^{hc/\lambda kT} - 1}$$

Wien's Law

$$\lambda_{peak}[nm] = \frac{2{,}900{,}000}{T[K]}$$

Color balancing

- Effect of different illuminants can be cancelled only in the spectral domain (impractical)
- Color balancing in 3-d color space is practical approximation
- Color constancy in human visual system: gain control in cone space LMS *[von Kries,1902]*
- Von Kries hypothesis applied to image acquisition devices (cameras, scanners)



- How to determine k_L, k_M, k_S automatically?

Color balancing (cont.)

- Von Kries hypothesis

$$\begin{pmatrix} L' \\ M' \\ S' \end{pmatrix} = \begin{pmatrix} k_L & 0 & 0 \\ 0 & k_M & 0 \\ 0 & 0 & k_S \end{pmatrix} \begin{pmatrix} L \\ M \\ S \end{pmatrix}$$

- If illumination (or a patch of white in the scene) is known, calculate

$$k_L = \frac{L_{desired}}{L_{actual}}; \quad k_M = \frac{M_{desired}}{M_{actual}}; \quad k_S = \frac{S_{desired}}{S_{actual}}$$

Color balancing with unknown illumination

- Gray-world

$$k_L \sum_{x,y} L[x,y] = k_M \sum_{x,y} M[x,y] = k_S \sum_{x,y} S[x,y]$$

- Scale-by-max

$$k_L \max_{x,y} L[x,y] = k_M \max_{x,y} M[x,y] = k_S \max_{x,y} S[x,y]$$

- Shades-of-gray
  [Finlayson, Trezzi, 2004]

$$k_L \left(\sum_{x,y} L^p[x,y] \right)^{\frac{1}{p}} = k_M \left(\sum_{x,y} M^p[x,y] \right)^{\frac{1}{p}} = k_S \left(\sum_{x,y} S^p[x,y] \right)^{\frac{1}{p}}$$

  » Special cases: gray-world ($p = 1$), scale-by-max ($p = \infty$)

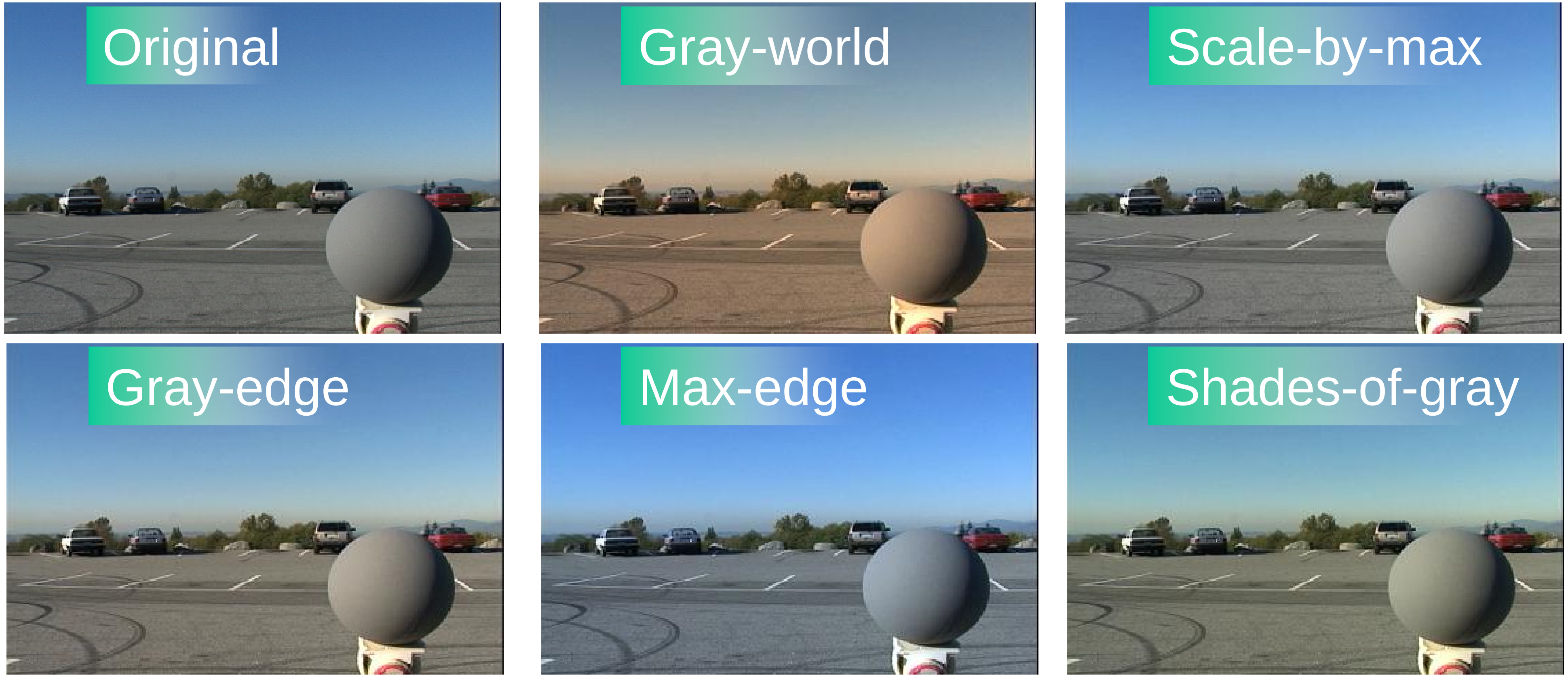
  » Best performance for $p \approx 6$

- Refinements:
  smooth image, exclude saturated color/dark pixels,
  use spatial derivatives instead ("gray-edge," "max-edge")
  [van de Weijer, 2007])

Color balancing example

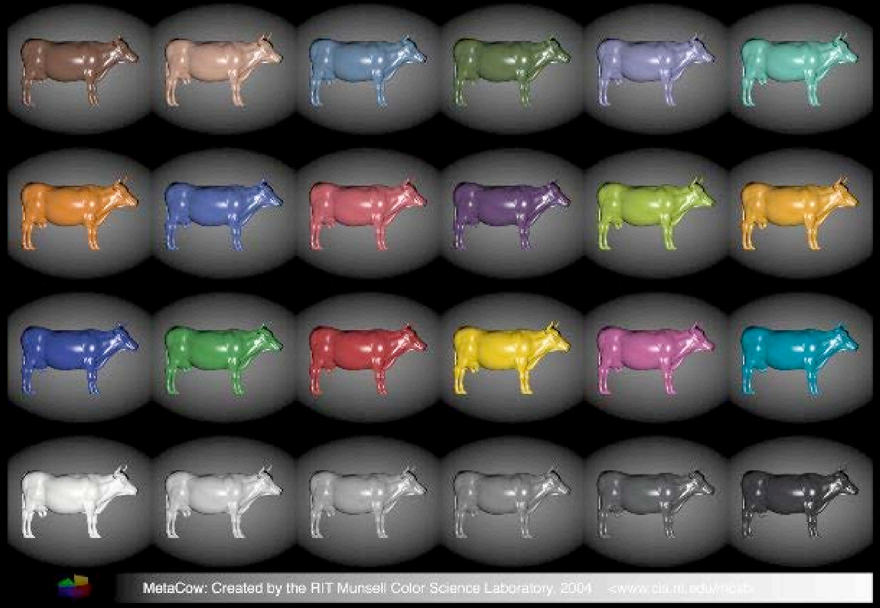
Original Gray-world Scale-by-max Gray-edge Max-edge Shades-of-gray

Color balancing example



Original image courtesy Ciurea and Funt

Daylight D65
CIE observer



Daylight D65
cheap camera

Illuminant A
CIE observer

Color conversion cheat sheet (e.g. for HW2)

- great website for insights, every possible color conversion scheme, and much more:
  www.brucelindbloom.com

- spectrum to CIE XYZ:
  (no illuminant)

$$X = \int_\lambda \bar{x}(\lambda) P(\lambda) d\lambda$$

$$Y = \int_\lambda \bar{y}(\lambda) P(\lambda) d\lambda$$

$$Z = \int_\lambda \bar{z}(\lambda) P(\lambda) d\lambda$$

CIE XYZ to CIE xyY:

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$Y = Y$$

- CIE XYZ to CIE RGB:

$$\begin{bmatrix} R_{linear} \\ G_{linear} \\ B_{linear} \end{bmatrix} = M^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

approximation of CIE gamma:

$$\{R,G,B\} = \{R,G,B\}_{linear}^{1/\gamma}$$

- CIE RGB to CIE XYZ:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = M \begin{bmatrix} R_{linear} \\ G_{linear} \\ B_{linear} \end{bmatrix} \qquad M = \begin{bmatrix} .490 & .310 & .200 \\ .177 & .813 & .011 \\ .000 & .010 & .990 \end{bmatrix}$$