Displacement estimation

- Displacement estimation by block matching
  - Search strategies
  - Subpixel estimation
Where is the defect?

Image $g[x,y]$ (no defect)  
Image $f[x,y]$ (w/ defect)
Absolute difference between two images

\[ |f-g| \text{ w/o alignment} \]

\[ |f-g| \text{ w/ alignment} \]
Displacement estimation by block matching

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match.

Image $g[x,y]$  

Image $f[x,y]$  

Rectangular array of pixels is selected as a measurement window.
Displacement estimation by block matching

\[ \text{Image } g[x, y] \quad \text{Image } f[x, y] \]

... process repeated for another measurement window position.
Integer pixel shifts

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match.

Rectangular array of pixels is selected as a measurement window.
Integer pixel shifts

Rectangular array of pixels is selected as a measurement window

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match
Error metric

- **Sum of Squared Differences**
  \[
  SSD[\Delta_x, \Delta_y] = \sum_{[x,y] \in \text{msmnt window}} (f[x,y] - g[x+\Delta_x, y+\Delta_y])^2
  \]

- Alternatives: SAD (Sum of Absolute Differences), cross correlation, mutual information . . .
- Robustness against outliers: sum of saturated squared differences, median of squared differences . . .
SSD values resulting from block matching

- Estimated displacement
- Integer-pixel accuracy
Block matching: search strategies

Full search

- All possible displacements within the search range are compared.
- Computationally expensive
- Highly regular, parallelizable
Conjugate direction search

- Alternate search in x and y directions
- Stop when there is no further improvement
Block matching: search strategies

Coarse-to-fine

- Start with coarsely spaced candidate displacements
- Smaller pattern when best match is in the middle
- Stop when desired displacement accuracy is reached
Block matching: search strategies

**Diamond search** [Li, Zeng, Liou, 1994] [Zhu, Ma, 1997]

- Start with large diamond pattern at [0,0]
- If best match lies in the center of large diamond, proceed with small diamond
- If best match does not lie in the center of large diamond, center large diamond pattern at new best match
Interpolation of the SSD Minimum

SSD

Horizontal shift $\Delta x$

Sub-pixel Accurate Minimum

Fit parabola through $> 3$ points approximately
2-d Interpolation of SSD Minimum

Paraboloid
- Perfect fit through 6 points
- Approximate fit through > 6 points

SSD

Vertical shift $\Delta y$

Horizontal shift $\Delta x$