Gray level histograms

Brain image

#pixels

0.5

1

1.5

2

2.5

3

3.5

4

$4 \times 10^4$

gray level
Gray level histogram in viewfinder
Gray level histograms

To measure a histogram:
- For B-bit image, initialize $2^B$ counters with 0
- Loop over all pixels $x, y$
- When encountering gray level $f[x,y]=i$, increment counter $#i$

Normalized histogram can be thought of as an estimate of the probability distribution of the continuous signal amplitude

Use fewer, larger bins to trade off amplitude resolution against sample size.
Histogram equalization

Idea:

Find a non-linear transformation

\[ g = T(f) \]

that is applied to each pixel of the input image \( f[x,y] \), such that a uniform distribution of gray levels results for the output image \( g[x,y] \).
Histogram equalization

Analyse ideal, continuous case first ...

Assume

- Normalized input values $0 \leq f \leq 1$ and output values $0 \leq g \leq 1$
- $T(f)$ is differentiable, increasing, and invertible, i.e., there exists

$$f = T^{-1}(g) \quad 0 \leq g \leq 1$$

**Goal:** pdf $p_g(g) = const$ over the entire range $0 \leq g \leq 1$
Histogram equalization for continuous case

- From basic probability theory

\[ p_f(f) \xrightarrow{f} T(f) \xrightarrow{g} p_g(g) = \left[ p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} \]

- Consider the transformation function

\[ g = T(f) = \int_0^f p_f(\alpha) d\alpha \quad 0 \leq f \leq 1 \]

- Then . . .

\[ \frac{dg}{df} = p_f(f) \]

\[ p_g(g) = \left[ p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} = \left[ p_f(f) \frac{1}{p_f(f)} \right]_{f=T^{-1}(g)} = 1 \quad 0 \leq g \leq 1 \]
Histogram equalization for discrete case

- Now, $f$ only assumes discrete amplitude values $f_0, f_1, \ldots, f_{L-1}$ with empirical probabilities

$$
P_0 = \frac{n_0}{n}, \quad P_1 = \frac{n_1}{n}, \quad \ldots \quad P_{L-1} = \frac{n_{L-1}}{n} \quad \text{where } n \text{ is total number of pixels}
$$

- Discrete approximation of $g = T(f) = \int_0^f p_f(\alpha) \, d\alpha$

$$
g_k = T \left[ f_k \right] = \sum_{i=0}^{k} P_i \quad \text{for } k = 0, 1, \ldots, L-1
$$

- The resulting values $g_k$ are in the range $[0,1]$ and might have to be scaled and rounded appropriately.
Histogram equalization example

Original image Bay

... after histogram equalization
Histogram equalization example

Original image *Bay* ... after histogram equalization

![Original histogram](image1.png)

![Equalized histogram](image2.png)
Histogram equalization example

Original image *Brain*  ... after histogram equalization
Histogram equalization example

Original image *Brain*

... after histogram equalization
Histogram equalization example

Original image *Moon*  ... after histogram equalization
Histogram equalization example

Original image *Moon*  . . . after histogram equalization

![Histogram of Original Image](image1)

![Histogram of EQUALIZED Image](image2)
Contrast-limited histogram equalization

![Diagram showing contrast-limited histogram equalization with input and output gray levels and histograms for different clipping levels.](image-url)
Adaptive histogram equalization

- Histogram equalization based on a histogram obtained from a portion of the image

- Sliding window approach: different histogram (and mapping) for every pixel

- Tiling approach: subdivide into overlapping regions, mitigate blocking effect by smooth blending between neighboring tiles

- Limit contrast expansion in flat regions of the image, e.g., by clipping histogram values. (“Contrast-limited adaptive histogram equalization”)

[Pizer, Amburn et al. 1987]
Adaptive histogram equalization

Original image
Parrot

Adaptive histogram equalization, 8x8 tiles

Global histogram equalization

Adaptive histogram equalization, 16x16 tiles
Adaptive histogram equalization

Original image
*Dental Xray*

Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles
Adaptive histogram equalization

Original image

**Skull Xray**

Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles
Ansel Adam’s Zone System (1939)

full tonal gradation

11-step gradation

<table>
<thead>
<tr>
<th>Zone</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Pure black</td>
</tr>
<tr>
<td>I</td>
<td>Near black, with slight tonality but no texture</td>
</tr>
<tr>
<td>II</td>
<td>Textured black; the darkest part of the image in which slight detail is recorded</td>
</tr>
<tr>
<td>III</td>
<td>Average dark materials and low values showing adequate texture</td>
</tr>
<tr>
<td>IV</td>
<td>Average dark foliage, dark stone, or landscape shadows</td>
</tr>
<tr>
<td>V</td>
<td>Middle gray: clear north sky; dark skin, average weathered wood</td>
</tr>
<tr>
<td>VI</td>
<td>Average Caucasian skin; light stone; shadows on snow in sunlit landscapes</td>
</tr>
<tr>
<td>VII</td>
<td>Very light skin; shadows in snow with acute side lighting</td>
</tr>
<tr>
<td>VIII</td>
<td>Lightest tone with texture: textured snow</td>
</tr>
<tr>
<td>IX</td>
<td>Slight tone without texture; glaring snow</td>
</tr>
<tr>
<td>X</td>
<td>Pure white: light sources and specular reflections</td>
</tr>
</tbody>
</table>

histogram of 11 zones → not quite flat