Introduction to color science

- Trichromacy
- Spectral matching functions
- CIE XYZ color system
- xy-chromaticity diagram
- Color gamut
- Color temperature
- Color balancing algorithms
Newton’s Prism Experiment - 1666
Color: visible range of the electromagnetic spectrum

380 nm 760 nm
Radiometry overview

illuminant $l(\lambda)$

spectral reflectance $\rho(x, y, \lambda)$ at receiver: $\rho(x, y, \lambda) \cdot l(\lambda)$
## Radiometric Quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Unit Description</th>
<th>Unit</th>
<th>Dimension</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiance</td>
<td>$L_e,\Omega$</td>
<td>watt per steradian per square metre</td>
<td>W·sr$^{-1}$·m$^{-2}$</td>
<td>M·T$^{-3}$</td>
<td>Radiant flux emitted, reflected, transmitted or received by a <em>surface</em>, per unit solid angle per unit projected area. This is a <em>directional</em> quantity. This is sometimes also confusingly called &quot;intensity&quot;.</td>
</tr>
<tr>
<td>Spectral radiance</td>
<td>$L_e,\Omega,\nu$ or $L_e,\Omega,\lambda$</td>
<td>watt per steradian per square metre per hertz or watt per steradian per square metre, per metre</td>
<td>W·sr$^{-1}$·m$^{-2}$·Hz$^{-1}$ or W·sr$^{-1}$·m$^{-3}$</td>
<td>M·T$^{-2}$ or M·L$^{-1}$·T$^{-3}$</td>
<td>Radiant flux of a <em>surface</em> per unit frequency or wavelength. The latter is commonly measured in W·sr$^{-1}$·m$^{-2}$·Hz$^{-1}$·nm$^{-1}$. This is a <em>directional</em> quantity. This is sometimes also confusingly called &quot;spectral intensity&quot;.</td>
</tr>
<tr>
<td>Irradiance</td>
<td>$E_\nu$</td>
<td>watt per square metre</td>
<td>W/m²</td>
<td>M·T$^{-3}$</td>
<td>Radiant flux <em>received</em> by a <em>surface</em> per unit area. This is sometimes also confusingly called &quot;intensity&quot;.</td>
</tr>
<tr>
<td>Spectral exposure</td>
<td>$N_{\nu}$ or $N_{\lambda}$</td>
<td>joule per square metre per hertz or joule per square metre, per metre</td>
<td>J/m²</td>
<td>M·T$^{-1}$ or M·L$^{-1}$·T$^{-2}$</td>
<td>Radiant energy received by a surface per unit area, or equivalently irradiance of a surface integrated over time of irradiation. This is sometimes also called &quot;radiant fluence&quot;.</td>
</tr>
<tr>
<td>Spectral exposure</td>
<td>$N_{\nu}$ or $N_{\lambda}$</td>
<td>joule per square metre per hertz or joule per square metre, per metre</td>
<td>J/m²·Hz$^{-1}$ or J/m²</td>
<td>M·T$^{-1}$ or M·L$^{-1}$·T$^{-2}$</td>
<td>Radiant exposure of a surface per unit frequency or wavelength. The latter is commonly measured in J·m$^{-2}$·nm$^{-1}$. This is sometimes also called &quot;spectral fluence&quot;.</td>
</tr>
</tbody>
</table>
## Photometric Quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Dimension</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminous energy</td>
<td>$Q_v$</td>
<td>lm·s</td>
<td>$T \cdot J$</td>
</tr>
<tr>
<td>Luminous flux / Luminous power</td>
<td>$\phi_v$</td>
<td>lm</td>
<td>$J$ [nb 3]</td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>$I_v$</td>
<td>cd</td>
<td>$J$ [nb 3]</td>
</tr>
<tr>
<td>Luminance</td>
<td>$L_v$</td>
<td>cd/m²</td>
<td>$L^{-2} \cdot J$</td>
</tr>
<tr>
<td>Illuminance</td>
<td>$E_v$</td>
<td>lx</td>
<td>$L^{-2} \cdot J$</td>
</tr>
<tr>
<td>Luminous exitance / Luminous emittance</td>
<td>$M_v$</td>
<td>lx</td>
<td>$L^{-2} \cdot J$</td>
</tr>
<tr>
<td>Luminous exposure</td>
<td>$H_v$</td>
<td>lx·s</td>
<td>$L^{-2} \cdot T \cdot J$</td>
</tr>
<tr>
<td>Luminous energy density</td>
<td>$\omega_v$</td>
<td>lumen second·per cubic metre</td>
<td>$L^{-3} \cdot T \cdot J$</td>
</tr>
<tr>
<td>Luminous efficacy</td>
<td>$\eta$ [nb 2]</td>
<td>lumen per watt</td>
<td>$M^{-1} \cdot L^{-2} \cdot T^3 \cdot J$</td>
</tr>
<tr>
<td>Luminous efficiency / Luminous coefficient</td>
<td>$V$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Human retina

[Roorda, Williams, 1999]

Pseudo-color image of nasal retina, 1 degree eccentricity, in two male subjects, scale bar 5 micron
Absorption of light in the cones of the human retina

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>445</td>
<td>L: S↓R (λ)</td>
</tr>
<tr>
<td>535</td>
<td>M: S↓G (λ)</td>
</tr>
<tr>
<td>575</td>
<td>S: S↓B (λ)</td>
</tr>
</tbody>
</table>

Note: curves are normalized. Much lower sensitivity to Blue, since fewer S-cones absorb less light.
Three-receptor model of color perception

- Different spectra can map into the same tristimulus values and hence look identical ("metamers")
- Three numbers suffice to represent any color – Grassmann’s law

\[
\begin{align*}
C(\lambda) & \quad \int S_R(\lambda) C(\lambda) d\lambda \quad \alpha_R \\
& \quad \int S_G(\lambda) C(\lambda) d\lambda \quad \alpha_G \\
& \quad \int S_B(\lambda) C(\lambda) d\lambda \quad \alpha_B
\end{align*}
\]

Effective cone stimulation ("tristimulus values")

[T. Young, 1802] [J.C. Maxwell, 1890]
Color matching

- Suppose 3 primary light sources with spectra $P_k(\lambda)$, $k = 1,2,3$
- Intensity of each light source can be adjusted by factor $\beta_k$
- How to choose $\beta_k$, $k = 1,2,3$, such that desired tristimulus values $(\alpha_R, \alpha_G, \alpha_B)$ result?

\[
C(\lambda) = \int_{\lambda} S_R(\lambda) C(\lambda) d\lambda
\]

\[
C(\lambda) = \beta_1 P_1(\lambda) + \beta_2 P_2(\lambda) + \beta_3 P_3(\lambda)
\]

\[
\alpha_i = \int_{\lambda} S_i(\lambda) \left[ \beta_1 P_1(\lambda) + \beta_2 P_2(\lambda) + \beta_3 P_3(\lambda) \right] d\lambda
\]

\[
= \beta_1 \cdot K_{i,1} + \beta_2 \cdot K_{i,2} + \beta_3 \cdot K_{i,3}
\]

with $K_{i,j} = \int_{\lambda} S_i(\lambda) P_j(\lambda) d\lambda$

Color matching is linear!
Additive vs. subtractive color mixing
Color matching experiment

[Diagram of a color matching experiment with primary lights, test light, surround field, and eyes.]

Courtesy B. Wandell, from [Foundations of Vision, 1996]
Spectral matching functions

- Color matching experiment: Monochromatic test light and monochromatic primary lights
- Spectral RGB primaries (scaled, such that R=G=B matches spectrally flat white)
- "Negative intensity": color is added to test color
- Standard human observer: CIE (Commision Internationale de L'Eclairage), 1931.

**RGB Color Matching Functions**

- \( r(\lambda) \)
- \( g(\lambda) \)
- \( b(\lambda) \)

### Wavelength \( \lambda \) (nm)

- 435.8 nm
- 546.1 nm
- 700.0 nm

### Tristimulus values

- 0
- 0.5
- 1
- 1.5
- 2
- 2.5
- 3
- 3.5

---

*Digital Image Processing: Bernd Girod, Gordon Wetzstein © 2013-2016 Stanford University -- Color 13*
Luminosity function

- **Experiment:**
  Match the brightness of a white reference light and a monochromatic test light of wavelength $\lambda$.
- **Links photometric to radiometric quantities**
CIE 1931 XYZ color system

Properties:
- All positive spectral matching functions
- Y corresponds to luminance
- Equal energy white: $X=Y=Z$
- Virtual primaries

XYZ Color Matching Functions

<table>
<thead>
<tr>
<th>Wavelength $\lambda$ (nm)</th>
<th>$x(\lambda)$</th>
<th>$y(\lambda)$</th>
<th>$z(\lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>400</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>600</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>700</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>800</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Tristimulus values

$x(\lambda) = \begin{pmatrix} 0.490 & 0.310 & 0.200 \end{pmatrix} R_\lambda$

$y(\lambda) = \begin{pmatrix} 0.177 & 0.813 & 0.011 \end{pmatrix} G_\lambda$

$z(\lambda) = \begin{pmatrix} 0.000 & 0.010 & 0.990 \end{pmatrix} B_\lambda$
Color gamut and chromaticity

\[
x = \frac{X}{X + Y + Z}
\]

\[
y = \frac{Y}{X + Y + Z}
\]
CIE chromaticity diagram
Perceptual non-uniformity of xy chromaticity

Just noticeable chromaticity differences (10X enlarged)

[MacAdam, 1942]
Color gamut

NTSC phosphors

R: $x=0.67$, $y=0.33$
G: $x=0.21$, $y=0.71$
B: $x=0.14$, $y=0.08$

Reference white:
$x=0.31$, $y=0.32$
Illuminant C
White at different color temperatures

The diagram shows the CIE 1931 color space with white points at different color temperatures. The x and y coordinates represent the chromaticity coordinates of the white light. The color temperature, $T_c$ (K), is indicated along the right boundary of the diagram, with values ranging from 1500 K to 25000 K. The diagram illustrates how the perceived color of white light changes with varying color temperatures.
Blackbody radiation

Planck’s Law, 1900

\[ B_T(\lambda) = \frac{2hc^2}{\lambda^5} \frac{e^{hc/\lambda kT} - 1}{e^{hc/\lambda kT} - 1} \]

Wien’s Law

\[ \lambda_{\text{peak}}[\text{nm}] = \frac{2,900,000}{T[\text{K}]} \]
Color balancing

- Effect of different illuminants can be cancelled only in the spectral domain (impractical)
- Color balancing in 3-d color space is practical approximation
- Color constancy in human visual system: gain control in cone space LMS \([\text{von Kries, 1902}]\)
- Von Kries hypothesis applied to image acquisition devices (cameras, scanners)

How to determine \(k_L, k_M, k_S\) automatically?
Color balancing (cont.)

- Von Kries hypothesis

\[
\begin{pmatrix}
L' \\
M' \\
S'
\end{pmatrix} =
\begin{pmatrix}
k_L & 0 & 0 \\
0 & k_M & 0 \\
0 & 0 & k_S
\end{pmatrix}
\begin{pmatrix}
L \\
M \\
S
\end{pmatrix}
\]

- If illumination (or a patch of white in the scene) is known, calculate

\[
k_L = \frac{L_{desired}}{L_{actual}}; \quad k_M = \frac{M_{desired}}{M_{actual}}; \quad k_S = \frac{S_{desired}}{S_{actual}}
\]
Color balancing with unknown illumination

- Gray-world

\[ k_L \sum_{x,y} L[x,y] = k_M \sum_{x,y} M[x,y] = k_S \sum_{x,y} S[x,y] \]

- Scale-by-max

\[ k_L \max_{x,y} L[x,y] = k_M \max_{x,y} M[x,y] = k_S \max_{x,y} S[x,y] \]

- Shades-of-gray [Finlayson, Trezzi, 2004]

\[ k_L \left( \sum_{x,y} L^p [x,y] \right)^{\frac{1}{p}} = k_M \left( \sum_{x,y} M^p [x,y] \right)^{\frac{1}{p}} = k_S \left( \sum_{x,y} S^p [x,y] \right)^{\frac{1}{p}} \]

- Special cases: gray-world \((p = 1)\), scale-by-max \((p = \infty)\)

- Best performance for \(p \approx 6\)

- Refinements:
  smooth image, exclude saturated color/dark pixels,
  use spatial derivatives instead (“gray-edge,” “max-edge”)
  [van de Weijer, 2007]
Color balancing example

Original  Gray-world  Scale-by-max  Gray-edge  Max-edge  Shades-of-gray
Color balancing example

Original

Gray-world

Scale-by-max

Gray-edge

Max-edge

Shades-of-gray

Original image courtesy Ciurea and Funt
Daylight D65
CIE observer

Illuminant A
CIE observer

Daylight D65
cheap camera
Color conversion cheat sheet (e.g. for HW2)

- great website for insights, every possible color conversion scheme, and much more: www.brucelindbloom.com

- spectrum to CIE XYZ:
  (no illuminant)
  \[
  X = \int \bar{x}(\lambda)P(\lambda)\,d\lambda \\
  Y = \int \bar{y}(\lambda)P(\lambda)\,d\lambda \\
  Z = \int \bar{z}(\lambda)P(\lambda)\,d\lambda
  \]

- CIE XYZ to CIE xyY:
  \[
  x = \frac{X}{X + Y + Z} \\
  y = \frac{Y}{X + Y + Z} \\
  Y = Y
  \]

- CIE XYZ to CIE RGB:
  \[
  \begin{bmatrix}
  R_{\text{linear}} \\
  G_{\text{linear}} \\
  B_{\text{linear}}
  \end{bmatrix}
  = M^{-1}
  \begin{bmatrix}
  X \\
  Y \\
  Z
  \end{bmatrix}
  \]

- approximation of CIE gamma:
  \[
  \{R,G,B\} = \left\{R,G,B\right\}_{\text{linear}}^{1/\gamma}
  \]

- CIE RGB to CIE XYZ:
  \[
  \begin{bmatrix}
  X \\
  Y \\
  Z
  \end{bmatrix}
  = M
  \begin{bmatrix}
  R_{\text{linear}} \\
  G_{\text{linear}} \\
  B_{\text{linear}}
  \end{bmatrix}
  \]

  \[
  M = \begin{bmatrix}
  .490 & .310 & .200 \\
  .177 & .813 & .011 \\
  .000 & .010 & .990
  \end{bmatrix}
  \]