Edge detection

- Gradient-based edge operators
  - Prewitt
  - Sobel
  - Roberts

- Laplacian zero-crossings

- Canny edge detector

- Hough transform for detection of straight lines

- Circle Hough Transform
Gradient-based edge detection

- Idea (continuous-space): local gradient magnitude indicates edge strength
  \[
  |\text{grad}(f(x, y))| = \sqrt{\left(\frac{\partial f(x, y)}{\partial x}\right)^2 + \left(\frac{\partial f(x, y)}{\partial y}\right)^2}
  \]

- Digital image: use finite differences to approximate derivatives

- Edge templates
Practical edge detectors

- Edges can have any orientation
- Typical edge detection scheme uses $K=2$ edge templates
- Some use $K>2$

```
s[x, y] \rightarrow e_1[x, y] \rightarrow e_2[x, y] \rightarrow \ldots \rightarrow e_K[x, y]
```

Combination, e.g.,
```
\sum_k |e_k[x, y]|^2
```
or
```
\text{Max}_k |e_k[x, y]|
```

```
edge magnitude \rightarrow M[x, y]
```

```
(\text{edge orientation}) \rightarrow \alpha[x, y]
```
Gradient filters (K=2)

Central Difference
\[
\begin{pmatrix}
0 & 0 & 0 \\
-1 & [0] & 1 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
0 & -1 & 0 \\
0 & [0] & 0 \\
0 & 1 & 0
\end{pmatrix}
\]

Roberts
\[
\begin{pmatrix}
[0] & 1 \\
-1 & 0
\end{pmatrix}
\begin{pmatrix}
[1] & 0 \\
0 & -1
\end{pmatrix}
\]

Prewitt
\[
\begin{pmatrix}
-1 & 0 & 1 \\
-1 & [0] & 1 \\
-1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
-1 & -1 & -1 \\
0 & [0] & 0 \\
1 & 1 & 1
\end{pmatrix}
\]

Sobel
\[
\begin{pmatrix}
-1 & 0 & 1 \\
-2 & [0] & 2 \\
-1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
-1 & -2 & -1 \\
0 & [0] & 0 \\
1 & 2 & 1
\end{pmatrix}
\]
Kirsch operator (K=8)
Prewitt operator example

Original
1024x710

Magnitudes of images filtered with:

\[
\begin{pmatrix}
-1 & 0 & 1 \\
-1 & 0 & 1 \\
-1 & 0 & 1 \\
\end{pmatrix}
\]
(log display)

\[
\begin{pmatrix}
-1 & -1 & -1 \\
0 & 0 & 0 \\
1 & 1 & 1 \\
\end{pmatrix}
\]
(log display)
Prewitt operator example (cont.)

Sum of squared horizontal and vertical gradients (log display)

threshold = 900  threshold = 4500  threshold = 7200
Sobel operator example

Sum of squared horizontal and vertical gradients (log display)

threshold = 1600

threshold = 8000

threshold = 12800
Roberts operator example

Original 1024x710

Magnitude of image filtered with
\[
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\]
(log display)

Magnitude of image filtered with
\[
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\]
(log display)
Roberts operator example (cont.)

Sum of squared diagonal gradients (log display)

threshold = 100  
threshold = 500  
threshold = 800
Edge orientation

Central Difference

\[
\begin{pmatrix}
0 & 0 & 0 \\
-1 & 0 & 1 \\
0 & 0 & 0 \\
\end{pmatrix}
\]

Roberts

\[
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
[1] & 0 \\
0 & -1 \\
\end{pmatrix}
\]

Gradient scatter plot
Edge orientation

Prewitt
$$\begin{pmatrix}
-1 & 0 & 1 \\
-1 & 0 & 1 \\
-1 & 0 & 1
\end{pmatrix}$$

Sobel
$$\begin{pmatrix}
-1 & 0 & 1 \\
-2 & 0 & 2 \\
-1 & 0 & 1
\end{pmatrix}$$

Gradient scatter plot
Edge orientation

5x5 “consistent”
gradient operator
[Ando, 2000]
Gradient consistency problem

Known gray value

Calculate this value using gradient field

Same result, regardless of path?
Laplacian operator

- Detect edges by considering second derivative

\[
\nabla^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}
\]

- Isotropic (rotationally invariant) operator
- Zero-crossings mark edge location

![Graphs of Edge Profile and Derivatives](image)
Approximations of Laplacian operator by 3x3 filter

\[
\begin{pmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & -8 & 1 \\
1 & 1 & 1
\end{pmatrix}
\]
Zero crossings of Laplacian

- Sensitive to very fine detail and noise ➔ blur image first
- Responds equally to strong and weak edges ➔ suppress zero-crossings with low gradient magnitude
Filtering of image with Gaussian and Laplacian operators can be combined into convolution with Laplacian of Gaussian (LoG) operator.

\[
\text{LoG}(x, y) = -\frac{1}{\pi\sigma^4} \left(1 - \frac{x^2 + y^2}{2\sigma^2}\right) e^{-\frac{x^2 + y^2}{2\sigma^2}}
\]
Discrete approximation of Laplacian of Gaussian

$$\sigma = \sqrt{2}$$

\[
\begin{array}{cccccccccccc}
0 & 0 & 1 & 2 & 2 & 2 & 1 & 0 & 0 \\
0 & 2 & 3 & 5 & 5 & 5 & 3 & 2 & 0 \\
1 & 3 & 5 & 3 & 0 & 3 & 5 & 3 & 1 \\
2 & 5 & 3 & -12 & -23 & -12 & 3 & 5 & 2 \\
2 & 5 & 0 & -23 & -40 & -23 & 0 & 5 & 2 \\
2 & 5 & 3 & -12 & -23 & -12 & 3 & 5 & 2 \\
1 & 3 & 5 & 3 & 0 & 3 & 5 & 3 & 1 \\
0 & 2 & 3 & 5 & 5 & 5 & 3 & 2 & 0 \\
0 & 0 & 1 & 2 & 2 & 2 & 1 & 0 & 0 \\
\end{array}
\]
Zero crossings of LoG

\[
\sigma = \sqrt{2} \quad \quad \sigma = 2\sqrt{2} \quad \quad \sigma = 4\sqrt{2} \quad \quad \sigma = 8\sqrt{2}
\]
Zero crossings of LoG – gradient-based threshold

\[
\sigma = \sqrt{2} \quad \sigma = 2\sqrt{2} \quad \sigma = 4\sqrt{2} \quad \sigma = 8\sqrt{2}
\]
Canny edge detector

1. Smooth image with a Gaussian filter
2. Approximate gradient magnitude and angle (use Sobel, Prewitt . . .)
   \[
   M[x, y] \approx \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}
   \]
   \[
   \alpha[x, y] \approx \tan^{-1}\left(\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}\right)
   \]
3. Apply nonmaxima suppression to gradient magnitude
4. Double thresholding to detect strong and weak edge pixels
5. Reject weak edge pixels not connected with strong edge pixels

[Canny, IEEE Trans. PAMI, 1986]
Canny nonmaxima suppression

- Quantize edge normal to one of four directions: horizontal, \(-45^\circ\), vertical, \(+45^\circ\)
- If \(M[x,y]\) is smaller than either of its neighbors in edge normal direction
  \(\Rightarrow\) suppress; else keep.

[Canny, IEEE Trans. PAMI, 1986]
Canny thresholding and suppression of weak edges

- Double-thresholding of gradient magnitude

  \[
  \begin{align*}
  \text{Strong edge:} & \quad M[x, y] \geq \theta_{\text{high}} \\
  \text{Weak edge:} & \quad \theta_{\text{high}} > M[x, y] \geq \theta_{\text{low}}
  \end{align*}
  \]

- Typical setting: \( \theta_{\text{high}}/\theta_{\text{low}} = 2...3 \)
- Region labeling of edge pixels
- Reject regions without strong edge pixels

[Canny, IEEE Trans. PAMI, 1986]
Canny edge detector

\[ \sigma = \sqrt{2} \quad \sigma = 2\sqrt{2} \quad \sigma = 4\sqrt{2} \]
Hough transform

- Problem: fit a straight line (or curve) to a set of edge pixels
- Hough transform (1962): generalized template matching technique
- Consider detection of straight lines $y = mx + c$
Hough transform (cont.)

- Subdivide \((m,c)\) plane into discrete “bins,” initialize all bin counts by 0
- Draw a line in the parameter space \([m,c]\) for each edge pixel \([x,y]\) and increment bin counts along line.
- Detect peak(s) in \([m,c]\) plane
Hough transform (cont.)

- Alternative parameterization avoids infinite-slope problem

\[ x \cos \theta + y \sin \theta = \rho \]

- Similar to Radon transform
Hough transform example

Original image
Hough transform example

Original image
Hough transform example

Original image
Hough transform example

De-skewed Paper

Global thresholding
Circle Hough Transform

- Find circles of fixed radius $r$

- Equivalent to convolution (template matching) with a circle
Circle Hough Transform for unknown radius

- 3-d Hough transform for parameters \((x_0, y_0, r)\)
- 2-d Hough transform aided by edge orientation ➔ “spokes” in parameter space

\[
\begin{align*}
&\text{edge pixel} \\
&\text{Circle center}
\end{align*}
\]
Example: circle detection by Hough transform

Original coins image

Prewitt edge detection

Detected circles