Gray level histograms

![Brain image](image)

Histogram

![Brain image](image)

- Gray level histograms
- Histograms
Gray level histograms

Bay image
Gray level histogram in viewfinder
Gray level histograms

- To measure a histogram:
  - For B-bit image, initialize $2^B$ counters with 0
  - Loop over all pixels $x,y$
  - When encountering gray level $f[x,y]=i$, increment counter $#i$

- Normalized histogram can be thought of as an estimate of the probability distribution of the continuous signal amplitude

- Use fewer, larger bins to trade off amplitude resolution against sample size.
**Histogram equalization**

**Idea:**

Find a non-linear transformation

\[ g = T(f) \]

that is applied to each pixel of the input image \( f[x,y] \), such that a uniform distribution of gray levels results for the output image \( g[x,y] \).
Histogram equalization

Analyse ideal, continuous case first ...

Assume

- Normalized input values $0 \leq f \leq 1$ and output values $0 \leq g \leq 1$
- $T(f)$ is differentiable, increasing, and invertible, i.e., there exists

$$f = T^{-1}(g) \quad 0 \leq g \leq 1$$

Goal: pdf $p_g(g) = 1$ over the entire range $0 \leq g \leq 1$
Histogram equalization for continuous case

- From basic probability theory
  \[ p_f(f) \xrightarrow{f}\ T(f) \xrightarrow{g} p_g(g) = \left[ p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} \]

- Consider the transformation function
  \[ g = T(f) = \int_0^f p_f(\alpha) d\alpha \quad 0 \leq f \leq 1 \]

- Then
  \[ \frac{dg}{df} = p_f(f) \]
  \[ p_g(g) = \left[ p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} = \left[ p_f(f) \frac{1}{p_f(f)} \right]_{f=T^{-1}(g)} = 1 \quad 0 \leq g \leq 1 \]
Histogram equalization for discrete case

- Now, $f$ only assumes discrete amplitude values $f_0, f_1, \ldots, f_{L-1}$ with empirical probabilities

  \[
  P_0 = \frac{n_0}{n}, \quad P_1 = \frac{n_1}{n}, \quad \ldots \quad P_{L-1} = \frac{n_{L-1}}{n}
  \]
  where $n$ is total number of pixels

- Discrete approximation of \( g = T(f) = \int_0^f p_f(\alpha) \, d\alpha \)

  \[
  g_k = T[f_k] = \sum_{i=0}^{k} P_i \quad \text{for} \quad k = 0, 1, \ldots, L - 1
  \]

- The resulting values $g_k$ are in the range $[0,1]$ and might have to be scaled and rounded appropriately.
Histogram equalization example

Original image *Bay*  ... after histogram equalization
**Histogram equalization example**

Original image *Bay* ... after histogram equalization

![Histogram equalization example](image-url)
Histogram equalization example

Original image *Brain*  
... after histogram equalization
Histogram equalization example

Original image Brain

... after histogram equalization

#pixels

gray level

#pixels

gray level

Histogram equalization example

Original image *Moon* ...

... after histogram equalization
Histogram equalization example

Original image *Moon* ... after histogram equalization

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<thead>
<tr>
<th>Gray level</th>
<th>#pixels</th>
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<tbody>
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<tr>
<td>200</td>
<td>8</td>
</tr>
<tr>
<td>250</td>
<td>10</td>
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<table>
<thead>
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<th>#pixels</th>
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<td>200</td>
<td>4</td>
</tr>
<tr>
<td>250</td>
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</tr>
</tbody>
</table>
Contrast-limited histogram equalization

Contrast-limited histogram equalization involves limiting the contrast of the output image to prevent clipping. The graph shows the input and output gray levels for different input gray levels, with the number of pixels at each gray level indicated. The graph illustrates how the output gray level is determined based on the input gray level, with the output level being limited to a maximum value to prevent clipping.

The images below demonstrate the effect of contrast-limited histogram equalization on an image of a moon. The leftmost image is the original image, and the rightmost image is the result of contrast-limited histogram equalization. The intermediate images show the effect of limiting the contrast to different values, with the graph indicating the number of pixels at each gray level for the input and output images.
Adaptive histogram equalization

- Histogram equalization based on a histogram obtained from a portion of the image

  Sliding window approach: different histogram (and mapping) for every pixel

  Tiling approach: subdivide into overlapping regions, mitigate blocking effect by smooth blending between neighboring tiles

- Limit contrast expansion in flat regions of the image, e.g., by clipping histogram values. ("Contrast-limited adaptive histogram equalization")

  [Pizer, Amburn et al. 1987]
Adaptive histogram equalization

Original image *Parrot*

Adaptive histogram equalization, 8x8 tiles

Global histogram equalization

Adaptive histogram equalization, 16x16 tiles
Adaptive histogram equalization

Original image
*Dental Xray*

Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles
Adaptive histogram equalization

Original image

Skull Xray

Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles