Gray level histograms

Brain image

Histogram

#pixels

gray level

\(4 \times 10^4\)
Gray level histograms

Bay image
Gray level histogram in viewfinder
Gray level histograms

To measure a histogram:
- For B-bit image, initialize $2^B$ counters with 0
- Loop over all pixels $x,y$
- When encountering gray level $f[x,y]=i$, increment counter $#i$

Normalized histogram can be thought of as an estimate of the probability distribution of the continuous signal amplitude

Use fewer, larger bins to trade off amplitude resolution against sample size.
**Histogram equalization**

**Idea:**

Find a non-linear transformation

\[
g = T(f)
\]

that is applied to each pixel of the input image \( f[x,y] \), such that a uniform distribution of gray levels results for the output image \( g[x,y] \).
Histogram equalization

Analyse ideal, continuous case first ...

Assume

- Normalized input values $0 \leq f \leq 1$ and output values $0 \leq g \leq 1$
- $T(f)$ is differentiable, increasing, and invertible, i.e., there exists

$$f = T^{-1}(g) \quad 0 \leq g \leq 1$$

Goal: pdf $p_g(g) = 1$ over the entire range $0 \leq g \leq 1$
Histogram equalization for continuous case

- From basic probability theory

\[ p_f(f) \xrightarrow{T(f)} g \]

\[ p_g(g) = \left[ p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} \]

- Consider the transformation function

\[ g = T(f) = \int_0^f p_f(\alpha) d\alpha \quad 0 \leq f \leq 1 \]

- Then...

\[ \frac{dg}{df} = p_f(f) \]

\[ p_g(g) = \left[ p_f(f) \frac{df}{dg} \right] = \left[ p_f(f) \frac{1}{p_f(f)} \right] = 1 \quad 0 \leq g \leq 1 \]
Histogram equalization for discrete case

- Now, $f$ only assumes discrete amplitude values $f_0, f_1, \ldots, f_{L-1}$ with empirical probabilities
  \[ P_0 = \frac{n_0}{n} \quad P_1 = \frac{n_1}{n} \quad \ldots \quad P_{L-1} = \frac{n_{L-1}}{n} \quad \text{where } n \text{ is total number of pixels} \]

- Discrete approximation of $g = T(f) = \int_{0}^{f} p_f(\alpha) \, d\alpha$

  \[ g_k = T[f_k] = \sum_{i=0}^{k} P_i \quad \text{for } k = 0, 1, \ldots, L - 1 \]

- The resulting values $g_k$ are in the range $[0,1]$ and might have to be scaled and rounded appropriately.
Histogram equalization example

Original image *Bay*  ... after histogram equalization
Histogram equalization example

Original image *Bay*  ... after histogram equalization

- **Original Image**: A grayscale image labeled "Bay".
- **Histogram**: The left graph shows the histogram with a peak at low gray levels, indicating a dark image. The right graph shows a flatter histogram after equalization, indicating a more balanced distribution of gray levels.

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Histogram equalization example

Original image *Brain*

... after histogram equalization
Histogram equalization example

Original image *Brain*  

... after histogram equalization

![Original image](image1)

![Histograms](image2)
Histogram equalization example

Original image *Moon*  
... after histogram equalization
Histogram equalization example

Original image *Moon* . . . after histogram equalization

<table>
<thead>
<tr>
<th>Gray level</th>
<th>Original image</th>
<th>. . . after histogram equalization</th>
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Contrast-limited histogram equalization

Contrast-limited histogram equalization is a technique used to improve the contrast of images without clipping the pixel values. It is achieved by limiting the output gray level values to those that are close to the input gray level values, thereby preserving the details of the image.

### Diagram

- **Input gray level** vs. **Output gray level**
- **#pixels** vs. **Input gray level**

**Graphs:**
- **No clipping**
- **Clipped values**

**Images:**
- Before and after contrast-limited histogram equalization
Adaptive histogram equalization

- Histogram equalization based on a histogram obtained from a portion of the image

- Sliding window approach:
  different histogram (and mapping) for every pixel

- Tiling approach:
  subdivide into overlapping regions, mitigate blocking effect by smooth blending between neighboring tiles

- Limit contrast expansion in flat regions of the image, e.g., by clipping histogram values.
  (“Contrast-limited adaptive histogram equalization”)

[Pizer, Amburn et al. 1987]
Adaptive histogram equalization

Original image
Parrot

Adaptive histogram equalization, 8x8 tiles

Global histogram equalization

Adaptive histogram equalization, 16x16 tiles
Adaptive histogram equalization

Original image
*Dental Xray*

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles

Global histogram equalization
Adaptive histogram equalization

Original image *Skull Xray*

Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles