A Proximal Algorithm for Total Variation-Based Phase Retrieval

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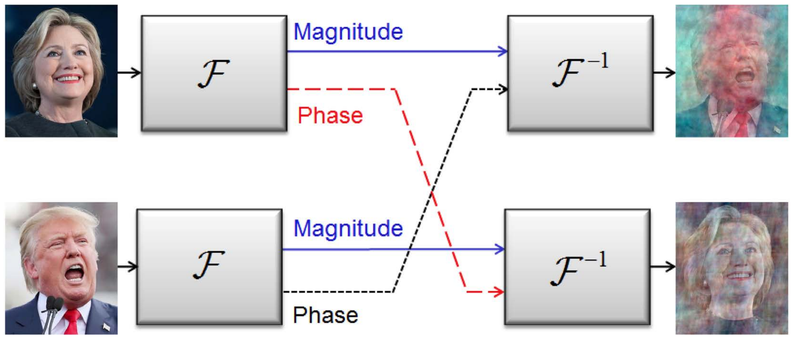
Motivation

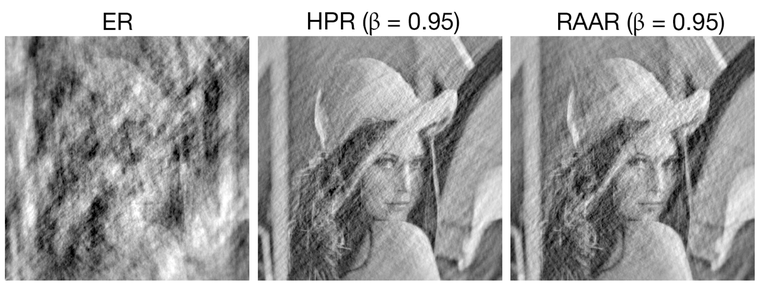
- In many applications, we can only measure the magnitude of the Fourier transform of a signal
- Examples: Optical imaging, X-ray crystallography, astronomy
- However, phase contains important information!



- **Given Fourier magnitude $b \equiv |\mathcal{F}v|$, recover signal v**

Related Work

- Projection algorithms
  - $\mathcal{P}_S$: Project into set of signals with same support
  - $\mathcal{P}_B$: Project into set of signals with same Fourier magnitude
  - Combine in weighted sum and iterate

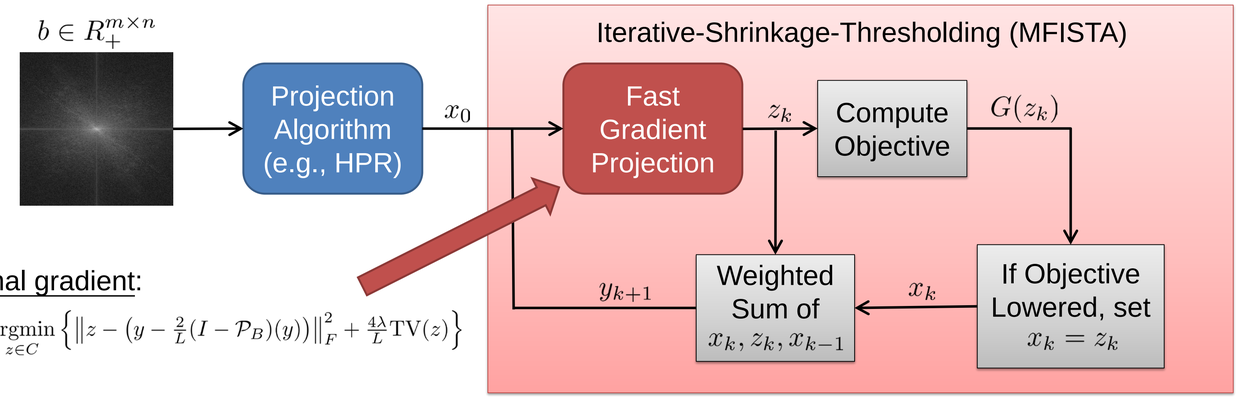


- Fienup (1982): Error Reduction (ER)
- Bauschke (2003): Hybrid Reflection Projection (HPR)
- Luke (2005): Relaxed Avg Alternating Reflection (RAAR)

Optimization with Total Variation Penalty

$$\min_{x \in C} \{G(x) \equiv \||\mathcal{F}x| - b\|_F^2 + 2\lambda \mathrm{TV}(x)\} \text{ where } \mathrm{TV}(x) = \|\nabla x\|_1$$

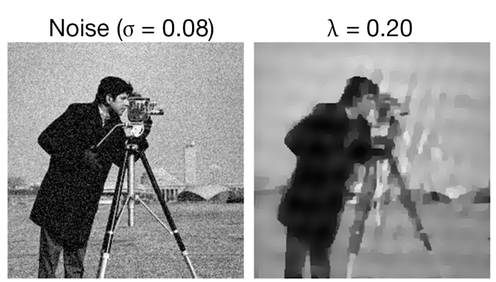
- TV penalizes difference between neighboring pixels, smoothing/denoising signal
- Solve with proximal gradient method using dual approach on convex sub-problem

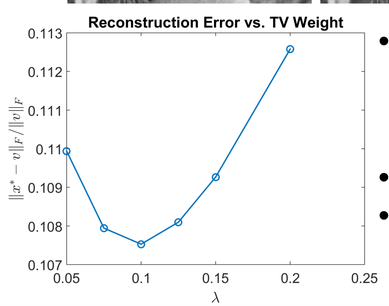


Proximal gradient:

$$p_L(y) = \underset{z \in C}{\operatorname{argmin}} \left\{ \left\| z - \left(y - \tfrac{2}{L}(I - \mathcal{P}_B)(y)\right) \right\|_F^2 + \tfrac{4\lambda}{L} \mathrm{TV}(z) \right\}$$

Experimental Results





- TV regularization reduces graininess, resulting in smaller reconstruction error

$$\mathrm{err}(x^*, v) = \frac{\|x^* - v\|_F}{\|v\|_F}$$

- Must search over parameters (λ, L)
- As noise grows, compensate by raising TV weight at cost of lower accuracy

Method	Lena	Camera
ER	0.3474	0.5670
HPR	0.1258	0.2781
RAAR	0.1372	0.3410
ISTPRA	**0.1075**	**0.2094**