PPAP: Phasing Process with *A Priori*

Po-Nan Li, Yen-Kai Huang and Felix Heide*

Department of {Electrical Engineering, Computer Science}, Stanford University // *project mentor

Abstract

Phase retrieval is an essential problem for coherent X-ray diffraction imaging (CDI), a lensless microscopic technique which measures an object's diffraction intensity [1]. The object's real-space image is generally inaccessible unless both its spectral magnitude and phase components are known. While CDI can theoretically provide high resolution image, and the phase can be in principal extracted from oversampled diffraction pattern, the spatial resolution of reconstructed image is limited by phase retrieval fidelity and quantum noise.

Here we propose to use a low-resolution image as *a priori* to facilitate the phase retrieval. Simulation results show that the prior image can serve as a stronger constraint than the conventionally used sample support. We report with our Phasing Process with A Priori (PPAP) method the phase retrieval can converge in just a few (<10) iterations, while other methods under comparison might take hundreds of iteration.

HIO

Traditional phase retrieval algorithms, e.g. HIO, start with random guess phase values. The algorithms then reconstruct and refine the image by moving the image back and forth in the real and reciprocal spaces, where constraints are applied to the immediate result [2]. Common constraints are (i) the fact that the sample has a finite size in real-space (enabled by oversampling) and (ii) replacing the calculated Fourier magnitude component with experimentally measured one (ground truth).

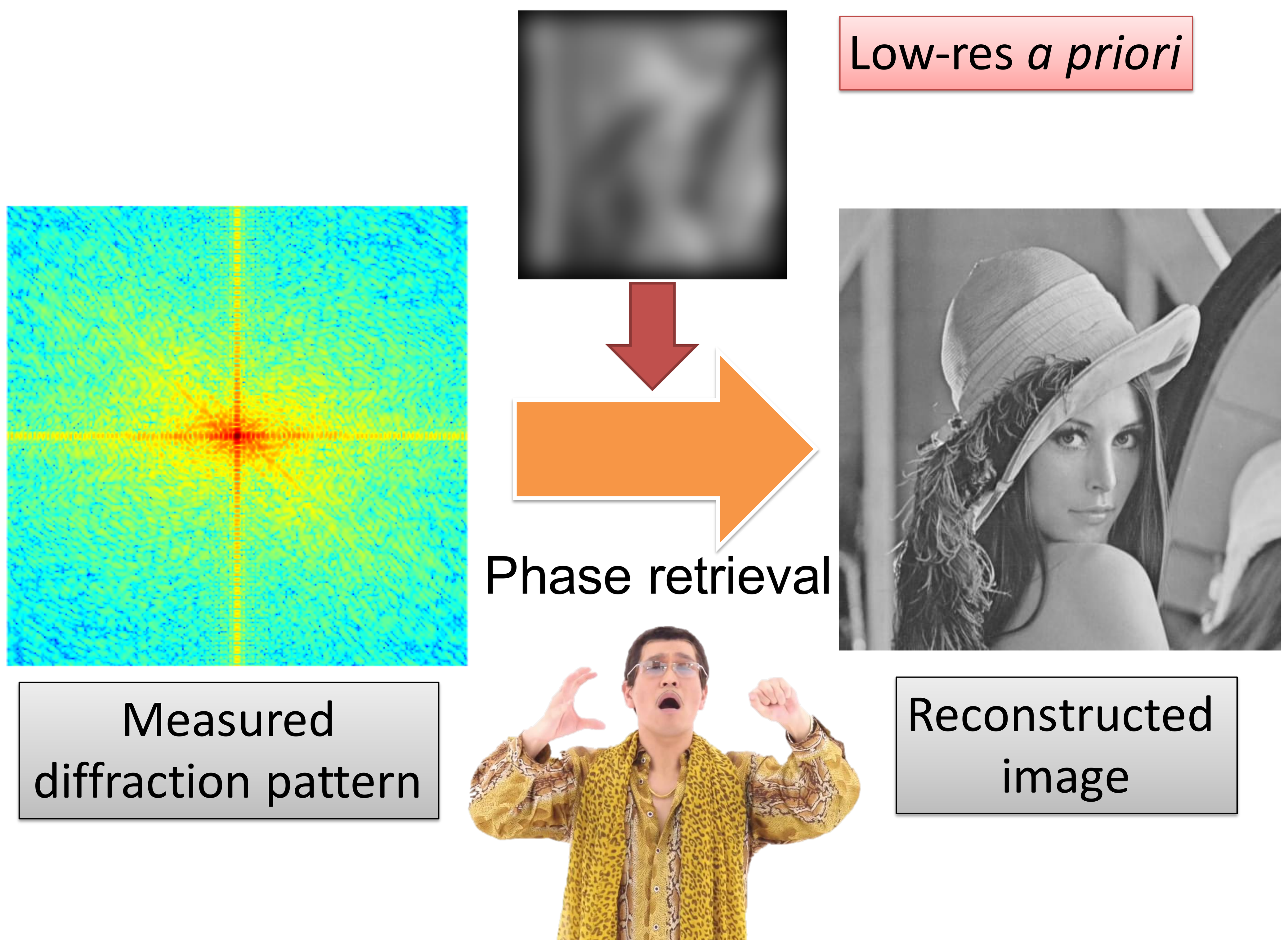
(i) $\mathcal{P}_{\mathcal{X}}(x) = \begin{cases} x(r), & \text{if } r \in D, \\ 0, & \text{otherwise,} \end{cases}$

(ii) $\mathcal{P}_{\mathcal{Y}}(x) = \mathcal{F}^*(\hat{y}), \quad \text{where } \hat{y} = \begin{cases} b(\omega)\frac{\hat{x}(\omega)}{|\hat{x}(\omega)|}, & \text{if } \hat{x}(\omega) \neq 0, \\ b(\omega), & \text{otherwise,} \end{cases}$

ADM

In order to have a more general framework for incorporating different constraints, we use the Alternating Direction Method (ADM), which allows us to formulate an optimization problem with two or more variables that are related by a Lagrangian function $\mathcal{L}(x, y, \lambda) := \lambda^\top (x - y) + \frac{1}{2}\|x - y\|^2$ [3]. By splitting λ from the update values, ADM is able to keep a balance between different constrains and not weighing one over another. ADM then solves the optimization problem with an iterative algorithm with following update equations:

$$x^{k+1} = \arg\min_{x \in \mathcal{X}} \mathcal{L}(x, y^k, \lambda^k),$$
$$y^{k+1} = \arg\min_{y \in \mathcal{Y}} \mathcal{L}(x^{k+1}, y, \lambda^k),$$
$$\lambda^{k+1} = \lambda^k + \beta(x^{k+1} - y^{k+1}),$$

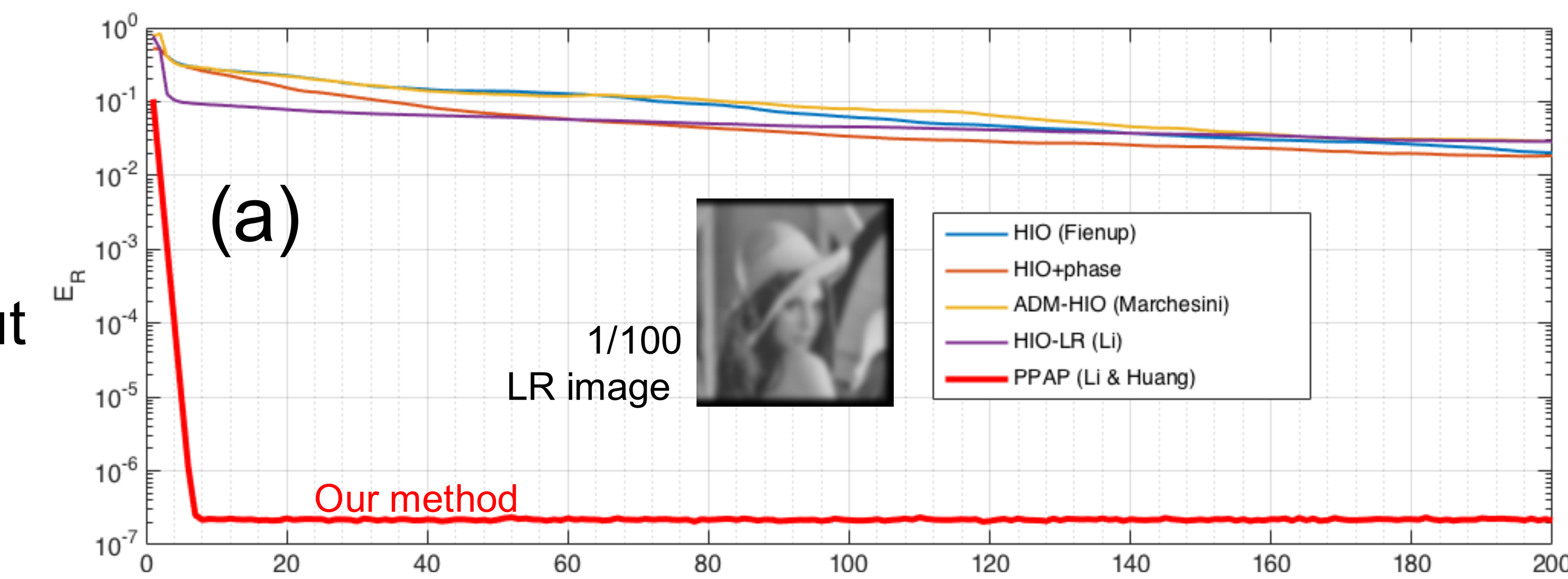


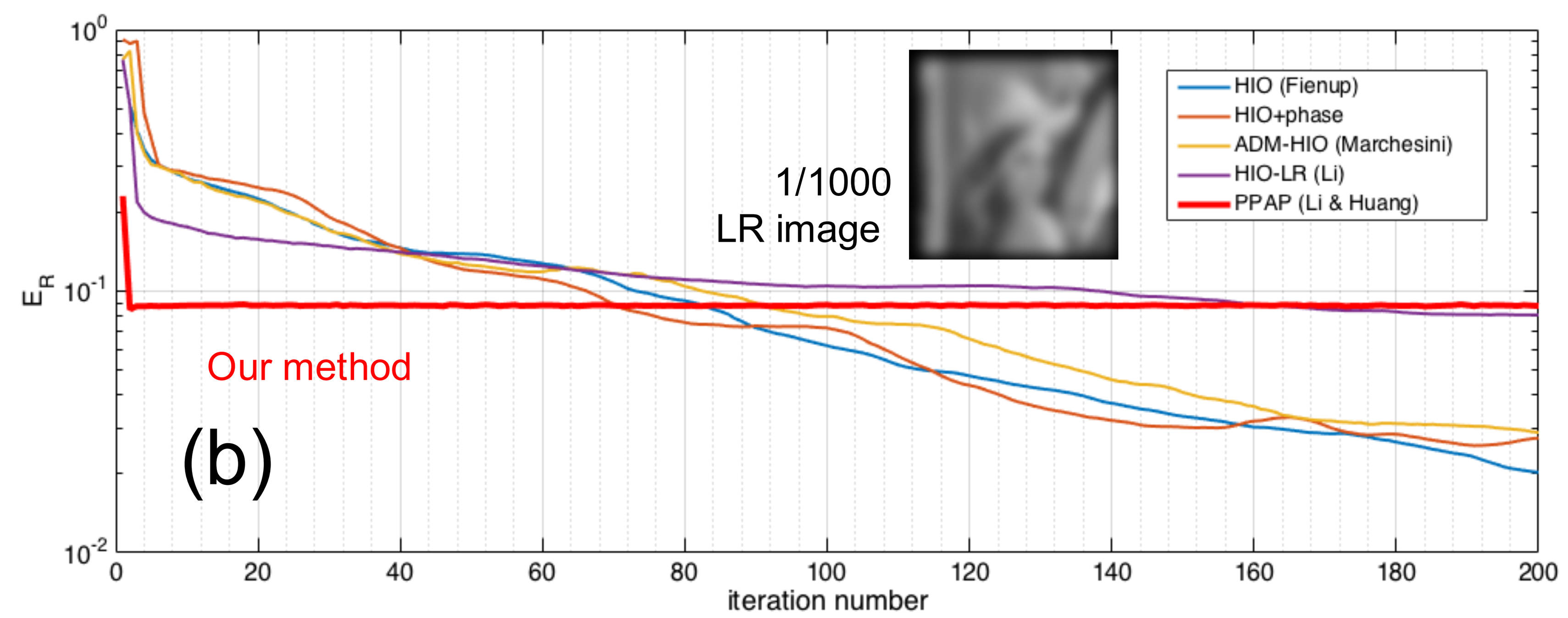
PPAP & Results

We took the ADM method and modified the update equation of ***x*** by replacing it with a low resolution (LR) image, denoted ***b***. We found out this algorithm works reasonably well.

$$x^{(t+1)} = b - \lambda^{(t)}$$
$$y^{(t+1)} = \mathcal{P}_{\mathcal{Y}}(x^{(t+1)} + \lambda^{(t)})$$
$$\lambda^{(t+1)} = \lambda^{(t)} + \beta(x^{(t+1)} - y^{(t+1)}).$$

Our method converges within < 10 steps while reaching incredibly low error, as long as a low-res image is given (a). Even if the LR quality is poor, our method can still serve as boosting for other method, e.g. HIO (b).





[1] J. Miao et al., *Nature* 400, 342 (1999). [2] R. Fienup, *Appl. Opt.* 21, 2758 (1982). [3] Z. Wen et al., *Inverse Prob.* 28, 115010 (2012).

Stanford University