

EE376A: Review Session 1

Channel Capacity

Recap: Strategies for finding Channel Capacity

- (a) Can you reduce the channel to ones which you have already seen? BSC, BEC etc?
- (b) Employ Symmetry of the channel?
- (c) Additive Independent Noise Channel
- (d) Is it over binary inputs? (direct optimization can be simple)
- (f) If you want to find the capacity in terms of other channels (say C_1, C_2, \dots), then go back to the definition and analyze the mutual information.
- (g) Use the operational definition of channel capacity

We will take a look at a few problems to emphasize these points:

1 Modulo Channel

- (a) Consider the DMC defined as follows: Output $Y = X \oplus_2 Z$ where X , taking values in $\{0, 1\}$, is the channel input, \oplus_2 is the modulo-2 summation operation, and Z is binary channel noise uniformly distributed over $\{0, 1\}$ and independent of X . What is the capacity of this channel?
- (b) Consider the channel of the previous part, but suppose that instead of modulo-2 addition $Y = X \oplus_2 Z$, we perform modulo-3 addition $Y = X \oplus_3 Z$. Now what is the capacity?
- (c) Now suppose the noise Z is no longer independent of the input X , but is instead described by the following conditional distribution:

$$p(Z = z|X = 0) = \begin{cases} 1/4 & \text{if } z = 0 \\ 3/4 & \text{if } z = 1, \end{cases}$$

and

$$p(Z = z|X = 1) = 1/2 \quad \text{both for } z = 0 \text{ and } z = 1.$$

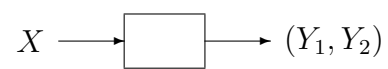
A random code of size 2^{nR} is generated uniformly (that is all codewords are drawn i.i.d. $X \sim \text{Bern}(0.5)$). Find the value V such that if $R < V$ then the average probability of decoding error (average both across the messages and the randomness in the codebook) vanishes with increasing blocklength while if $R > V$ then it does not.

2 A channel with two independent looks at Y.

Let Y_1 and Y_2 be conditionally independent and conditionally identically distributed given X . Thus $p(y_1, y_2|x) = p(y_1|x)p(y_2|x)$.

(a) Show $I(X; Y_1, Y_2) = 2I(X; Y_1) - I(Y_1; Y_2)$.

(b) Conclude that the capacity of the channel



is less than twice the capacity of the channel

