Review 2 solutions

March 19, 2018

1. See solution of problem 3, this can be solved in a similar way.
2. See Example 11.5.1 in Cover and Thomas textbook.
3. We use theorem 11.4.1 (Sanov's Theorem) from the textbook with the set E being the set of probability distributions satisfying $P_A = 2P_B$. Then the exponent is given by

$$\min_{P \in E} D(P||Q)$$

where Q is the uniform distribution on $\{A, B, C, D, E, F\}$. Since Q is uniform, we have $D(P||Q) = -H(P) + \log 6$. Thus minimizing the KL divergence is equivalent to maximizing $H(P)$ subject to the constraint $P_A = 2P_B$. Let $P_B = x$. Then $P_A = 2x$. For maximizing the entropy, all other symbols (C, D, E, F) should have equal probability, which is $(1 - 3x)/4$. Thus the entropy is given by

$$H(P) = -x \log x - 2x \log 2x - (1 - 3x) \log \frac{1 - 3x}{4}$$

We can maximize this wrt x (by differentiation) to get x^* and P^*. Then the exponent is given by $D(P^*||Q) = -H(P^*) + \log 6$.