EE376A: Review Session 2

Method of Types

1 Recap

The method is types is a very useful concept, used to understand the problems involving i.i.d sequences. The most important point to understand is that, we cluster together sequences with the same empirical statistics together into a type. Although the total number of sequences ($|\mathcal{X}|^n$) are exponential in n, the total types are polynomial in n ($\leq (n+1)^{|\mathcal{X}|}$). This has a huge impact on the analysis. Important basic properties of types include (it is important to understand these inequalities, and also how we obtained them):

1. Let x^n, y^n be two sequences in a type $\mathcal{T}(P)$. Then, for any function $f(x)$:
$$\frac{1}{n}\sum_{i=1}^{n} f(x_i)p(x_i) = \frac{1}{n}\sum_{i=1}^{n} f(y_i)p(y_i) = E_P(f(X))$$
2. Total number of types is $\leq (n+1)^{|\mathcal{X}|}$
3. Size of a type is bounded by:
$$(n+1)^{-|\mathcal{X}|}2^{nH(p)} \leq |\mathcal{T}(P)| \leq 2^{nH(p)}$$
4. (Sanov Theorem) Let X_i be distributed i.i.d according to the distribution P. Then:
$$P\left(\frac{1}{n}\sum_{i=1}^{n} X_i > \alpha\right) \doteq 2^{-n \min_{E_Q[X]>\alpha} D(Q||P)}$$

It is also important to understand the definitions of Strongly typical sets and their differences with weakly typical sets. The definition is useful for the achievability part of the rate distortion theorem.

1. $P(\mathcal{T}_\delta(X)) \to 1$ as $n \to \infty$
2. Typical Average lemma.

We will take a look at some simple problems where these definitions are useful.

2 Problems

1. Suppose we do a coin toss experiment with n fair coin tosses. Find the probability (correct in the exponent) of obtaining number of Tails, twice the number of Heads.
2. Suppose we toss a fair die n times. What is the probability that average of the throws is greater than 4 (correct in the exponent)?
3. Suppose we toss a die with state A, B, C, D, E, F, n times. What is the probability that we observe number of A, twice the number of observed B's.