EE 378A Statistical Signal Processing Professor Andrea Montanari

Homework 2

Due on: 04/25/2022

Instructions: this homework has a numerical component. Please submit your code and the simulation results together with your solution.

Reading

Section 7.7 from *Exercise 7.9 from S. Mallat, A Wavelet Tour of Signal Processing*

Problem 1

Exercise 7.9 from S. Mallat, A Wavelet Tour of Signal Processing
Let θ be a box spline of degree m obtained by $m+1$ convolutions of $\mathbf{1}_{[0,1]}$ with itself.

(a) Prove that

$$\theta(t) = \frac{1}{m!} \sum_{k=0}^{m+1} (-1)^k \binom{m+1}{k} ([t-k]_+)^m,$$

where $[x]_+ = \max(x, 0)$. *Hint:* Write $\mathbf{1}_{[0,1]} = \mathbf{1}_{[0,+\infty]} + \mathbf{1}_{(1,+\infty)}$

(b) Let A_m and B_m be the Riesz bounds of $\{\theta(t-n)\}_{n\in\mathbb{Z}}$. Prove that $\lim_{m\to+\infty} B_m = +\infty$ using the fact that $\{\theta(t-n)\}_{n\in\mathbb{Z}}$ is a Riesz basis if and only if (theorem 3.4 and equation 7.9 from the book)

$$\forall \omega \in [-\pi, \pi], \quad A \leq \sum_{k\in\mathbb{Z}} |\hat{\theta}(\omega + 2\pi k)|^2 \leq B.$$

Compute numerically A_m and B_m for $m \in \{0, \dots, 5\}$. You are welcome to use your favourite programming language.

Problem 2

Image mosaic
Exercise 7.27 from S. Mallat, A Wavelet Tour of Signal Processing
Let $f_0[n_1, n_2]$ and $f_1[n_1, n_2]$ be two square images of N pixels. We want to merge the center of $f_0[n_1, n_2]$ for $N^{1/2}/4 \leq n_1, n_2 < 3N^{1/2}/4$ in the center of f_1. For two grayscale images of your choice compute numerically the wavelet coefficients of f_0 and f_1. At each scale 2^j and orientation $1 \leq k \leq 3$ replace the $2^{-2j}/4$ wavelet coefficients corresponding to the center of f_1 by the wavelet coefficients of f_0. Reconstruct an image from this manipulated wavelet representation. Explain why the image f_0 seems to be merged in f_1, , without the strong boundary effects that are obtained when directly replacing the pixels of f_1 by the pixels of f_0. *Feel free to use whatever programming language you feel comfortable with, both MATLAB and Python have wavelet packages.*