The final consists in designing a rate-compatible code for the binary symmetric channel, for a prescribed set of rates, and error probability.

The objective is to maximize the amount of noise tolerated.

Constraints

The noisy channel is the binary symmetric channel, with crossover probability $\epsilon$, $\text{BSC}(\epsilon)$. The output alphabet is $Y = \{0, 1\}$, the input is $X = \{0, 1\}$, and the transition probability density

$$Q(y|x) = \begin{cases} 1 - \epsilon & \text{if } x = y, \\ \epsilon & \text{otherwise}. \end{cases}$$

Our aim is to design a rate compatible linear code for the three design rates $R = \{1/2, 2/3, 3/4\}$. More explicitly, we want three codes with generating matrices $G_1, G_2, G_3$, with dimensions $N_1 \times n$, $N_2 \times n$, $N_3 \times n$ such that:

1. As usual, codeword $x_a \in \{0, 1\}^{N_a}$, of the $a$-th code is generated from the information vector $z$ via

$$x_a = G_a z \mod 2.$$  

2. The code rates are $n/N_1 = 1/2$, $n/N_2 = 2/3$, $n/N_3 = 3/4$, or equivalently, the block-lengths are $N_1 = 2n$, $N_2 = (3/2)n$, $N_3 = (4/3)n$.

3. $G_2$ is the submatrix formed by the first $(3/2)n$ rows of $G_1$, and $G_3$ is the submatrix formed by the first $(4/3)n$ rows of $G_1$.

The code must achieve bit error rate $P_b \leq 10^{-4}$ on this channel. The objective is the to maximize, for each rate in the set $R$, the channel flip $\epsilon$ probability for which this constraint can be satisfied. In other words, defining $\epsilon_a$ has the maximum noise level for which code $G_a$ achieves $P_b \leq 10^{-4}$, we want to maximize $\epsilon_1, \epsilon_2, \epsilon_3$.

Notice that the code block-length is not constrained.

Presentation of the results

You are required to write a short report on your design. Unless you do not agree, the reports will be posted on the course webpage.

Reports must have the following form:


2. A complete description of the code. For instance if you are using any matrix in your design (like the parity check matrix in binary input channels and LDPC) you might submit the matrix along with your report (as a file).

3. A description of the decoding algorithm.
4. A print-out of the program used for simulations.

5. Three plots of simulation results reporting empirical bit error rate, for the three codes $G_1, G_2, G_3$. In each case, you need to plot the empirical $P_b$ on a grid of values $\epsilon \in [0.8 \cdot \epsilon_a, 1.2 \cdot \epsilon_a]$ (including in particular $\epsilon_a$ itself).

It must convincingly show that the claimed bit error probability is indeed achieved.

Reports are due by Sunday, March 6 at 12 noon Pacific time. Each group will give a short presentation (20 minutes) of their ideas and results in front of the class on Monday, March 7 (morning).