Branch and Bound Examples
Example from control theory

- function minimized is complicated, non-convex function that arises in control theory (stability degree)
- lower and upper bound evaluated using sophisticated methods (convex optimization, control theory)
- problem instance has 5 variables
Figure 1: Upper and lower bound versus number of iterations.
Figure 2: Pruned volume versus number of iterations.
Figure 3: Active rectangles versus number of iterations.
• approximately 1000 iterations to prune 82\% of the volume.

• approximately 2000 iterations to find global solution within 0.001
Example of mixed Boolean-convex problem

We consider the problem

\[
\begin{align*}
\text{minimize} & \quad c^T z \\
\text{subject to} & \quad d^T z \leq -1, \\
& \quad z_j \in \{0, 1\}, \quad j = 1, \ldots, n,
\end{align*}
\]

where

\[
c = (1.1, -2.2, 3.4, -3.7, 6), \quad d = (-1.1, -2.2, -3.4, 3.2, 5)
\]

(easily solved; this is just to illustrate branch and bound)
• split node with lowest lower bound

• fix variable closest to either 0 or 1 in the relaxation, or with largest dual variable among those equal to 0 or 1

• algorithm terminates in 10 steps (cf. 32 for exhaustive search)
\begin{itemize}
  \item $z_4 = 0$
  \item $z_4 = 1$
  \item $z_5 = 0$
  \item $z_5 = 1$
  \item $z_3 = 0$
  \item $z_3 = 1$
  \item $z_2 = 0$
  \item $z_2 = 1$
  \item $z_1 = 0$
  \item $z_1 = 1$
\end{itemize}

**Figure 4:** Branch and bound algorithm for Boolean problem.

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