Optimal Transceiver Design for Multi-Access Communication

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Main Points

An important problem in the management of communication networks: resource allocation

A goal: high data rate, low bit error rate

Transmitter + receiver for multi-user high-speed broadband Digital Subscriber Line

Frequency and transmitting power: goal

Further simplification to SOC, and to combinatorial polynomial time algorithm

Direct formulation is nonconvex; equivalent formulation is SDP (thus convex);

Valuable guidelines and insights for optimal practical transceiver design
Elements of data communication: OFDM, subcarriers, precoding/equalization.

Contet
is the additive (Gaussian) noise

are transmitter filter and equalizer filter respectively

is a linear, time-invariant channel (assumed known)

is input signal (assumed statistically white)
An Equivalent MIMO System

\[ x = H^*p + s, \quad H^* = \text{circulant} \]

\[ H: \text{channel matrix (obtained from } f), \quad n: \text{noise} \]

\[ G: u \times f \text{ receiver equalizer matrix (obtained from } g) \]

\[ F: u \times g \text{ transmitter matrix filter (or precoder), obtained from } f; \text{ data rate } u/g \]

\[ S/P: \text{serial-parallel converter (with cyclic prefixing)}; \quad P/S: \text{parallel-serial} \]
Multi-Input Multi-Output Communication System

Single-Input Single-Output Communication System

\( H^* \) is a circulant matrix

\( X \) received signal

\( s \)
The circulant channel matrix $H$ can be diagonalized via IFFT/FFT: $H^* = D_H^*H$.

Orthogonal Frequency Division Multiplexing (OFDM) system employs IFFT/FFT to decompose the channel $H^*$.

This diagonalization is not channel dependent.

Each subchannel corresponds to a subcarrier with a particular frequency (from FFT).

The diagonalized channel becomes a set of independent subchannels where $D_H$ is the standard discrete Fourier transform matrix, $H$ is diagonal.

OFDM System
Orthogonal Frequency Division Multiplexing (OFDM) System

Multi-Input Multi-Output Communication System

Equivalent to a Diagonal Channel Matrix
The precoder $F$ can be a general $n \times n$ matrix, subject to power constraint $\text{tr}(FF^\top) \leq p$. 

A special, and popular, linear precoder is the so called \textit{power loading precoder}: $F$ is diagonal. The optimized design $F$ will have a rank $\frac{n}{\sqrt{c}}$, resulting in an optimal data rate $\frac{c}{n}$. The precoder $F$ can be a \textit{general} $u \times u$ matrix, subject to power constraint $\text{tr}(FF^\top) \leq p$. 

Linearly Precoded/Power Loaded OFDM
Linearly Precoded OFDM System

**Power-Loaded OFDM System**

**General Linearly Precoded OFDM System**

Linearly Precoded/Power Loaded OFDM
Two-User Multi-Access Communication Channel

Mathematical model:
\[ x = H_1 F_1 s_1 + H_2 F_2 s_2 + \frac{1}{2} n. \]

Goal: Given the channel matrices, \( H_1, H_2 \), design transceivers \( F_1, F_2, G_1, G_2 \).

Linear detection: \( s_i = \text{sign}(C_i^T x) \) with \( i = 1, 2 \), \( d > 0 \).

Diagram of Two-user Communication System
Applications and Previous Work

Applications include the current and future systems of DSL, DAB, DVB.

Equalizer (or receiver) design for a fixed transmitter has been researched extensively in the last two decades.

The joint transmitter and receiver (transceiver) design was considered recently, but only for the single user case.

The last two require complex receiver structures.

- channel capacity
- maximum information rate
- Minimum Mean Square Error
- Minimum Mean Square Error

In the single user transceiver design work, the design criteria used include:

- Minimum Mean Square Error
- Maximum Information Rate
- Minimum Mean Square Error
- Channel Capacity

Applications include the current and future systems of DSL, DAB, DVB.
The detection with receiver (equalizer) $G_i$:

\[ s_i = \text{sign}(G_i x). \]

Let $e_i$ denote the error vector (before making the hard decision) for user $i$, $i = 1, 2$.

The detection with receiver (equalizer) $G_i$: $s_i = \text{sign}(G_i x)$.

**Mean Square Error**
Our goal is to design a set of transmitting matrix filters \( F \) and a set of matrix equalizers \( G \) such that the total mean squared error is minimized.

\[ \text{MSE} = \text{tr}(F^\top e e^\top F) + \text{tr}(G^\top e e^\top G) \]

As is always the case in practice, there are power constraints on the transmitting matrix:

\[ \text{tr}(F^\top F) \leq p_1; \quad \text{tr}(G^\top G) \leq p_2 \]

The above is nonconvex.

We first eliminate the variables \( G \), the MSE equalizers.

\[ \exists d \geq (\text{tr}(F^\top F) + \text{tr}(F^\top G^\top G))^{-1} \text{tr}(F^\top F) \begin{bmatrix} \text{tr}(F^\top e e^\top F) \\ \text{tr}(F^\top e e^\top G) \end{bmatrix} \]

Similarly, the transmitting matrices are minimized.

\[ \text{MSE} \geq \text{tr}(F^\top F) \begin{bmatrix} \text{tr}(F^\top F) \\ \text{tr}(F^\top G^\top G) \end{bmatrix} \]

The Formulation: MMSE Equalizer Case
Formulation: MMSE Equalizer Case

By minimizing $E(e_1 e_y^1)$ with respect to $G_1$, we obtain the following MMSE equalizer for user 1:

$$G_1 = F_{y_1} H_{y_1} W$$

where

$$W = \begin{pmatrix} H_1^T F_{y_1} & \frac{1}{2} \mathbb{I}_{2} \end{pmatrix}$$

Substituting this into $E(e_1 e_y^1)$ gives:

$$E(e_1 e_y^1) = F_{y_1} H_{y_1} W$$

Similarly, the MMSE equalizer $G_2$ for user 2 is given by $G_2 = F_{y_2} H_{y_2} W$ and resulting minimized (with respect to $G_2$) mean square error for user 2 is given by $E(e_2 e_y^2)$:

$$E(e_2 e_y^2) = F_{y_2} H_{y_2} W$$

We obtain the following MMSE equalizer for user 2:

$$G_2 = F_{y_2} H_{y_2} W$$

where

$$W = \begin{pmatrix} H_2^T F_{y_2} & \frac{1}{2} \mathbb{I}_{2} \end{pmatrix}$$

By minimizing $E(e_1 e_y^1)$, we obtain the following MMSE equalizer for user 1:
Total MSE

where the last step follows from the definition of $W$.

\[ u + \text{tr}(W) \]
Formulation: MMSE Equalizer Case

Introduce matrix variables:

\[ U_1 = F_1^T F y_1; \]
\[ U_2 = F_2^T F y_2. \]

Then the MMSE transceiver design problem becomes

\[
\begin{aligned}
&\text{minimize} & & U_1; U_2 \\
&\text{subject to} & & \text{tr}(U_1) \leq p_1; \]
\[ & & & \text{tr}(U_2) \leq p_2; \]
\[ & & & U_1 \succeq 0; \]
\[ & & & U_2 \succeq 0.
\end{aligned}
\]

Reformulate using the auxiliary matrix variable \( W \):

\[
\begin{aligned}
&\text{minimize} & & W; U_1; U_2 \\
&\text{subject to} & & \text{tr}(U_1) \leq p_1; \]
\[ & & & \text{tr}(U_2) \leq p_2; \]
\[ & & & W \succeq (H_1 U_1 H_1^T + H_2 U_2 H_2^T + \frac{1}{2} I) \succeq 0; \]
\[ & & & U_1 \succeq 0; \]
\[ & & & U_2 \succeq 0.
\end{aligned}
\]

Then the MMSE transceiver design problem becomes

\[
\begin{aligned}
0 & \preceq W \preceq 1; \\
0 & \preceq U_1 \preceq 1; \\
0 & \preceq U_2 \preceq 1,
\end{aligned}
\]

subject to

\[
\begin{aligned}
\text{tr}(W) & \geq (\text{tr}(W))^T; \\
\text{tr}(U_1) & \geq 0; \\
\text{tr}(U_2) & \geq 0;
\end{aligned}
\]

Introduce matrix variables:

\[
\begin{aligned}
H_2 W_2 H_2^T & = H_2^T; \\
H_1 W_1 H_1^T & = H_1^T.
\end{aligned}
\]
The constraint
\[ W \preceq H_1 U_1 H_1^T + H_2 U_2 H_2^T + \frac{1}{2} I \]
is equivalent to LMI:
\[ \begin{bmatrix} I & H_1 \Omega H_2^T \\ H_2 \Omega H_1^T & \mathbb{W} \end{bmatrix} \succeq 0. \]

We obtain an SDP formulation:
\[ \begin{array}{c}
\text{minimize} \\
W; U_1; U_2
\end{array} \]
subject to
\[ \begin{bmatrix} I & H_1 \Omega H_2^T \\ H_2 \Omega H_1^T & \mathbb{W} \end{bmatrix} \succeq 0. \]

Interior point method with arithmetic complexity
\[ O \left( n^6 \log \frac{1}{\epsilon} \right), \quad \epsilon > 0 \]
is the solution accuracy.

SDP Formulation
OFDM: Diagonal Designs are Optimal!

Result

If $H_1$ and $H_2$ are diagonal, as in the OFDM systems, then the optimal transmitters are also diagonal.

Implication

The MMSE transceivers for an multi-user OFDM system can be implemented by optimally setting the data rates and allocating power to each subcarrier for all the users.
Linearly Precoded OFDM System

Power-Loaded OFDM System

General Linearly Precoded OFDM System
Restricting to diagonal designs, the SDP becomes SOC:

\[
\begin{align*}
\minimize_{w, u} & \quad 0 < w_i < 0 \quad \forall i \\
\text{subject to} & \quad \sum_{i=1}^n u_i (i) \cdot p_1, \\
& \quad \sum_{i=1}^n u_i (i) \cdot p_2, \\
& \quad w_i - j_1 (i) u_1 (i) + j_2 (i) u_2 (i) + \frac{1}{2} \leq 0, \\
& \quad u_1 (i) \geq 0, \quad u_2 (i) \geq 0, \quad i = 1, \ldots, n.
\end{align*}
\]

Arithmetic complexity \( O(n^{3.5} \log(n/e)) \) \( \varepsilon \) is the accuracy.

From SDP to SOC Formulation

Second order cone program.

There exist highly efficient (general purpose) interior point methods to solve the above.
\begin{align*}
\exists \xi \in (\mathcal{S} \setminus I) : \exists \xi \in (\mathcal{S} \setminus I)\quad &\text{we have } nI \cap \xi \supseteq \xi I \supseteq \xi I \cap \xi I, \text{ and any } nI \cap \xi I \supseteq \xi I \supseteq \xi I \cap \xi I, \text{ and any } nI \cap \xi I \supseteq \xi I \supseteq \xi I \cap \xi I. \\
\exists \xi \in (\mathcal{S} \setminus I) : \exists \xi \in (\mathcal{S} \setminus I)\quad &\text{we have } nI \cap \xi \supseteq \xi I \supseteq \xi I \cap \xi I, \text{ and any } nI \cap \xi I \supseteq \xi I \supseteq \xi I \cap \xi I. \\
\exists \xi \in (\mathcal{S} \setminus I) : \exists \xi \in (\mathcal{S} \setminus I)\quad &\text{we have } nI \cap \xi \supseteq \xi I \supseteq \xi I \cap \xi I. \\
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\exists \xi \in (\mathcal{S} \setminus I) : \exists \xi \in (\mathcal{S} \setminus I)\quad &\text{we have } nI \cap \xi \supseteq \xi I \supseteq \xi I \cap \xi I. \\
\exists \xi \in (\mathcal{S} \setminus I) : \exists \xi \in (\mathcal{S} \setminus I)\quad &\text{we have } nI \cap \xi \supseteq \xi I \supseteq \xi I \cap \xi I.
\end{align*}
\[
\begin{align*}
\mathbf{H}_1 \mathbf{F}_1 \mathbf{s}_1 + \mathbf{H}_2 \mathbf{F}_2 \mathbf{s}_2 + \frac{1}{2} \mathbf{n}, \text{with } \mathbf{H}_i, \mathbf{F}_i \text{ diagonal; } \\
\mathbf{x}(i) = \mathbf{h}_1(i) \mathbf{f}_1(i) \mathbf{s}_1(i) + \mathbf{h}_2(i) \mathbf{f}_2(i) \mathbf{s}_2(i) + \frac{1}{2} \mathbf{n}(i).
\end{align*}
\]

In a fading environment, the path gains \( j \mathbf{h}_1(i) j'^2 \) \( j \mathbf{h}_2(i) j'^2 \) are random, \( \mathbf{I}_s \) is singleton: at most one subcarrier should be shared by the two users. The remaining subcarriers are allocated to the two users according to the path gain ratios: subcarrier \( i \) to user 1 and subcarrier \( j \) to user 2 only if

\[
\frac{|(i) \mathbf{z}_q|}{|j \mathbf{y}_q|} < \frac{|(i) \mathbf{z}_q|}{|j \mathbf{y}_q|}.
\]

In a fading environment, the path gains are random:

\[
\mathbf{H} = \mathbf{u} \mathbf{d} + \mathbf{z} \mathbf{s} \mathbf{y} \mathbf{H} + \mathbf{l} \mathbf{s} \mathbf{y} \mathbf{H}^\dagger = \mathbf{x}.
\]

Intuitive Interpretation
Then $I_1 \supseteq I_2 \supseteq \ldots \supseteq I_n$ leads to an $O(n^3)$ strongly polynomial time (combinatorial) algorithm vs. $O(n^3 \log \frac{1}{\epsilon})$. Assume

$|\frac{(u)^T \eta}{u^T \eta}| < |\frac{(I - u)^T \eta}{(I - u)^T \eta}| < \ldots < |\frac{(z)^T \eta}{z^T \eta}| < |\frac{(I)^T \eta}{I^T \eta}|$

The properties of optimal MMSE transceivers can be used to design a combinatorial algorithm.
Practical Implications
Mathematical model:
\[
x = H_1 F_1 s_1 + H_2 F_2 s_2 + \cdots + H_m F_m s_m + \frac{1}{2} n
\]

Let \( G_i \) be the linear MMSE matrix equalizer at the \( i \)-th receiver. Then the total MSE problem can be described as:

\[
\begin{align*}
\min \quad & \operatorname{tr} \left( \left( I_d + \frac{1}{2} H_i F_i s_i \right) \right) \\
\text{subject to} \quad & \operatorname{tr} \left( G_i \right) \leq \frac{1}{2} p_i; \quad G_i \succeq 0
\end{align*}
\]

This is given by:

Let \( C_i \) be the linear MMSE matrix equalizer at the \( i \)-th receiver. Then the total MSE is:

\[
\begin{align*}
\min \quad & \operatorname{tr} \left( \left( I_d + \frac{1}{2} H_i F_i s_i \right) \right) \\
\text{subject to} \quad & \operatorname{tr} \left( G_i \right) \leq \frac{1}{2} p_i; \quad G_i \succeq 0
\end{align*}
\]
SOC Formulation

\[
\begin{array}{c}
\minimize_w \\
\text{subject to} \\
\end{array}
\]

SDP Formulation

\[
\begin{array}{c}
\minimize_{\mathbf{X}} \\
\text{subject to} \\
\end{array}
\]

\[
\begin{bmatrix}
\mathbf{I} & \mathbf{E} & \mathbf{W} & \cdots & \mathbf{W} \\
\mathbf{E} & \mathbf{I} & \mathbf{W} & & \mathbf{W} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\mathbf{W} & \mathbf{W} & \cdots & \mathbf{I} & \mathbf{E} \\
\end{bmatrix}
\]
but the average number of bits per block remains 10. The subcarrier allocations and the number of bits per block vary from block to block.

Simulation Scenario

3. Multi-user MSE power-loaded OFDM – Using the SOC Formulation. In this case, channels and does MSE power loading for these bits. Each user sends 1 bit per subcarrier, i.e. 10 bits per block; knows its allocated subcarrier allocation as AMOUR.

2. Individually MSE power-loaded OFDM – Same subcarrier allocation as AMOUR.

1. AMOUR – No channel knowledge; Each user uses 10 subcarriers, spreads 8 bits over these carriers using a DFT-type spreading.

Three schemes:

- Each user "sees" its own Rayleigh channel (complex-valued)
- Uplink with 16 active users and 160 available subcarriers
- Each user "sees" its own Rayleigh channel (complex-valued)
Simulation Results

16-user OFDM in a length 3 Rayleigh channel, 160 subcarriers, Coding gain smaller here

Average BER

BlockSNR dB

Single-user MSE power loading via SOCP, 10 bits/blk
AMOUR, no channel knowledge, 10 bits/blk
Multi-user MSE power loading via SOCP, 10 bits/blk on eve

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Efficiency of the Design Approach

On a PIII 600Mhz PC,

- SDP ~ 0.65 secs

- SOCP ~ 0.65 secs

16 users, 10 symbols per block, length 3 channel

- SDP ~ 0.13 secs

- SOCP ~ 0.13 secs

2 users, 2 symbols per block, length 3 channel

- SDP ~ 0.65 secs

- SOCP ~ 0.65 secs

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A convex problem can be solved by interior point methods, iterative water-filling:

\[
\text{maximize } \log \det (I + \sum_{k=1}^{K} H_k^H X_k H_k)
\]

subject to:

\[
\begin{align*}
\text{tr} (X_k) &\leq p_k \quad \forall k \in \{1, 2, \ldots, K\} \\
X_k &\succeq 0 \quad \forall k \in \{1, 2, \ldots, K\}
\end{align*}
\]

The multi-user transmitter design is then

which is achievable by successive nulling and cancellation at BS.

The total sum rate of multi-access channel is

\[
\log \det (X) + \log \det (I + \sum_{k=1}^{K} H_k^H X_k H_k)
\]

The total sum rate of multi-access channel is

\[
\sum_{k=1}^{K} \log \det (X_k) + \log \det (I + \sum_{k=1}^{K} H_k^H X_k H_k)
\]

\[
\begin{align*}
\text{tr} (X_k) &\leq p_k \quad \forall k \in \{1, 2, \ldots, K\} \\
X_k &\succeq 0 \quad \forall k \in \{1, 2, \ldots, K\}
\end{align*}
\]

\[
\text{Let } d_k \text{ denote the covariance matrix and the transmit power of the } k\text{-th user}
\]

Formulation: Max Sum Rate Capacity for MAC
Formulation: Max Sum Rate Capacity for BC

A convex problem: can be solved by interior point methods (and iterative water-filling).

\[ \text{Let } k \geq 0, \text{ denote the covariance matrix for the } k\text{-th user signal, and let } p \text{ denote the total transmit power.} \]

\[ \text{By duality, total sum rate of a broadcast channel channel is} \]

\[ \text{subject to} \]

\[ \log \det (I + \frac{p}{K} \sum_{k=1}^{K} H_k^H H_k) \]

\[ \text{maximize} \]

\[ \log \det (I + \frac{p}{K} \sum_{k=1}^{K} H_k^H H_k) + 1 \]

The multi-user transmitter design is then

\[ \text{which is achievable by dirty paper coding technique, where } \zeta_k \text{ (new variables) depends} \]

\[ \text{on } \zeta_k \text{ linearly.} \]

A convex problem; can be solved by interior point methods (and iterative water-filling).

\[ \text{Let } \zeta_k \geq 0, \text{ denote the covariance matrix for the } k\text{-th user signal, and let } p \text{ denote} \]

\[ \text{the total transmit power.} \]
Consider the zero-forcing equalizers:

\[ G_1 = (H_1 F_1)^{-1} \]
\[ G_2 = (H_2 F_2)^{-1} \]

Moreover,

\[ \mathbb{E}(e_1 e_y^1) = (G_1 H_1 F_1 - I) (G_1 H_1 F_1 - I) y + (G_1 H_1 F_2) (G_1 H_1 F_2) y + \frac{1}{2} G_1 y \]
\[ \mathbb{E}(e_2 e_y^2) = (G_2 H_2 F_2 - I) (G_2 H_2 F_2 - I) y + (G_2 H_1 F_1) (G_2 H_1 F_1) y + \frac{1}{2} G_2 y \]

Recall \( e \) denotes the error vector (before making the hard decision) for user \( i \), \( i = 1, 2 \).
\[ \frac{z}{1} H \frac{z}{1} H = \frac{z}{1} I \quad \frac{z}{1} H \frac{z}{1} H = \frac{z}{1} I \]

The ZF condition reduces to \( \frac{z}{1} I \) \( \frac{z}{1} H \frac{z}{1} H \) \( \frac{z}{1} I \) \( \frac{z}{1} I \)

\[ \frac{z}{1} d \geq (\frac{z}{1} H) \text{tr} \quad \frac{z}{1} d \geq (\frac{z}{1} H) \text{tr} \]

The power constraint becomes \( \text{tr} (\frac{z}{1} H) \text{tr} \).

\[ \text{MSE} = \text{tr} (\frac{z}{1} H) \text{tr} \]

Then the MSE can be rewritten as

\[ \text{MSE} = \text{tr} (\frac{z}{1} H) \text{tr} \]

Introduce new matrix variables

\[ \begin{align*}
L & = \frac{z}{1} C \\
U & = \frac{z}{1} C \\
V & = \frac{z}{1} C \\
W & = \frac{z}{1} C
\end{align*} \]

Substituting the ZF conditions into the MSE expressions gives

**Formulation: Zero-Forcing Equalizer Case**
The Minimum MSE transceiver design problem can be cast as

\[
\begin{align*}
\text{minimize} & \quad \text{MSE} = \text{tr}(V_1^H H_2^* U_2^H y_2) + \frac{1}{2} \text{tr}(V_1^H) + \text{tr}(V_2^H H_1^* U_1^H y_1) + \frac{1}{2} \text{tr}(V_2^H) \\
\text{subject to} & \quad \text{tr}(U_1) \cdot p_1 \\
& \quad \text{tr}(U_2) \cdot p_2 \\
& \quad V_{1i} \geq 0 \\
& \quad U_{1i} \geq 0 \\
& \quad i = 1, 2 \\
& \quad \text{MSE} = (\tilde{\Lambda})^d d + (\tilde{\Lambda} H \Lambda)^d d + (\tilde{\Lambda})^d d + (\tilde{\Lambda} H \Lambda)^d d \\
& \quad \text{subject to} \quad \text{tr}(\tilde{\Lambda}) \geq (\tilde{\Lambda} H \Lambda)^d d + (\tilde{\Lambda})^d d + (\tilde{\Lambda})^d d + (\tilde{\Lambda} H \Lambda)^d d \\
& \quad \text{MSE} = (\tilde{\Lambda})^d d + (\tilde{\Lambda} H \Lambda)^d d + (\tilde{\Lambda})^d d + (\tilde{\Lambda} H \Lambda)^d d
\end{align*}
\]

Reformulation is necessary.

- The objective function is nonconvex quadratic, due to the cross terms \( \text{tr}(V_1^H H_2^* U_2^H y_2) \) and \( \text{tr}(V_2^H H_1^* U_1^H y_1) \). Note the constraints are nonlinear (due to the matrix inverse).

The Minimum MSE transceiver design problem can be cast as

For formulation: Zero-Forcing Case
Reformulation: ZF Case

Use monotonicity and Schur complement technique, we obtain the following equivalent formulation:

\[
\begin{align*}
\text{minimize} & \quad \text{MSE} = \text{tr}(V_1^H V_2 + V_2^H V_1) + \frac{1}{2} \text{tr}(V_2^H V_2) \\
\text{subject to} & \quad \text{tr}(U_1) \cdot p_1 ; \text{tr}(U_2) \cdot p_2 ; V_i \succcurlyeq 0 ; U_i \succcurlyeq 0 ; i = 1, 2.
\end{align*}
\]

Note that the constraints are all linear matrix inequalities (LMIs), and in particular convex.

But the objective function is nonconvex.

\[
\begin{bmatrix}
\tilde{z} A & \tilde{z} \\
\tilde{z} & \tilde{z} H \tilde{z} H
\end{bmatrix}
\prec 0 \quad \text{and}
\begin{bmatrix}
\tilde{z} A & \\
\tilde{z} H & \tilde{z} H^T H
\end{bmatrix}
\prec 0
\]

\[\text{subject to} \quad \text{tr}(V_1^H V_2 + V_2^H V_1) = \text{MSE} \quad \text{and} \quad \text{tr}(V_2^H V_2) \geq (\tilde{z} \Omega) \text{tr} \quad \text{and} \quad \text{tr}(U_1) \cdot p_1 ; \text{tr}(U_2) \cdot p_2 ; V_i \succcurlyeq 0 ; U_i \succcurlyeq 0 ; i = 1, 2.
\]
AlternatingDirectionMethod

Fixing the designs for user 1 (namely, $U_1$, $V_1$), the objective function MSE is linear in $U_2$, $V_2$, resulting in a SDP. Similarly, fixing $U_2$, $V_2$ yields a semidefinite program.

Fixing the designs for user 1 (namely, $U_1$), the objective function MSE is linear in $U_2$, $V_2$, resulting in a SDP. Similarly, fixing $U_2$, $V_2$ yields a semidefinite program.

**AlternatingDirectionMethod**

\[ \text{Repeat with } \gamma := \gamma + 1. \]

- \begin{align*}
\text{At iteration } & I, \text{ let } \Lambda = (0) \Lambda = (0) \Lambda \\
\text{Update } & U_i \text{ and } V_i \text{ to the resulting optimized values of } \Lambda \text{ and } Z. \end{align*}

\[ \text{Solve } (1) \text{ with } U_i \text{ and } V_i \text{ fixed to the values of } U_i^{(k-1)} \text{ and } V_i^{(k-1)}. \]

\[ \text{Solve } (1) \text{ with } U_i \text{ and } V_i \text{ fixed to the values of } U_i \text{ and } V_i. \]

\[ \text{Update } U_i \text{ and } V_i \text{ to the resulting optimized values of } \Lambda \text{ and } Z. \]

\[ \text{At iteration } \gamma \leq I, \text{ set } \lambda := I. \]

\[ \text{Convergence: bounded iterates + the minimum principle necessary optimality condition.} \]
Letchannelmatrices $H_1$, $H_2$ bediagonal.

If we fix $U_2$, $V_2$ at some positive definite diagonal matrices in ($1$) and optimize with respect to $U_1$, $V_1$, then the resulting optimized matrices can also be taken to be positive definite and diagonal.

Conjecture: the optimal solutions of ($1$) are always diagonal.

The proof uses reduction and a property of bipartite matching polytope.

Let channel matrices $H_1$, $H_2$ be diagonal.

Imply the power-loaded OFDM is optimal.
The dual of (2) is a linearly constrained entropy maximization problem.

\[ \text{minimize} \sum_{i=1}^{n} v_1(i) \sum_{j=1}^{2} \Lambda_{ij} + v_2(i) \sum_{j=1}^{2} \Lambda_{ij} \]
subject to \[ u \leq (\gamma)_{1}^\Lambda (\gamma)_{2}^\Lambda - \sum_{i=1}^{n} \gamma_d \geq (\gamma)_{1}^\Lambda (\gamma)_{2}^\Lambda \]

The formulation (1) reduces to a geometric program.

Restrict to diagonal designs \( \cup_i \Lambda \text{ diagonal} \)
\[
\begin{align*}
\max & \quad \sum_{i=1}^{\infty} \frac{1}{\log i} \\
\text{s.t.} & \quad \sum_{i=1}^{\infty} \frac{1}{\log i} = c_i
\end{align*}
\]

where the coefficients \(c_i\) are defined as
\[
\begin{align*}
c_i &= \frac{1}{2} \log \left( 1 + \frac{1}{i} \right) - \frac{1}{2} \\
&= \frac{1}{2} \log \left( \frac{1 + \frac{1}{i}}{1 + \frac{1}{i-1}} \right)
\end{align*}
\]
So far we have

Future work

- Extension to the multi-user downlink case.
- Incorporating QoS and other receiver structures in the formulation.

Results provide valuable guidelines and insights for the practical system design.

- Demonstrated the potential of SDP/SOC/interior point methods in digital communication.
- Studied the properties of the optimal transceiver designs.
- Presented various SDP/SOC formulations and algorithms for the optimal transceiver design problems.

Summary