

EE469B: Assignment 5

Due Thursday Oct. 31

1. Spinor Magnitude Constraint Starting from the expressions for α and β as a function of a rotation angle θ about a unit vector $\mathbf{n} = (n_x, n_y, n_z)$, show that

$$\alpha\alpha^* + \beta\beta^* = 1$$

2. Composite Pulses Often combinations of pulses are used to perform specific tasks more accurately. One example is the sequence consisting of a rotation of θ about the x axis, followed by a rotation by the same θ about the y axis. Our goal in this case is to start from equilibrium $\mathbf{M} = (0, 0, M_0)$ and to accurately make M_z zero. The magnetization is left somewhere in the transverse plane, but we don't care where.

a) Solve for α and β for this pulse sequence.

b) Find a simple expression for the longitudinal magnetization. Solve for the θ that produces an $M_z = 0$. Plot the M_z as a function of θ for $0 < \theta < \pi$. What range of θ will result in M_z being less than $0.05M_0$?

c) Find a simple expression for the transverse magnetization. Plot M_{xy} as a function of θ for $0 < \theta < \pi$.

d) Plot $\angle M_{xy}$ as a function of θ for $0 < \theta < \pi$. Note that this function is approximately linear about $\theta = \pi/2$. Hence, what this composite pulse does is trade off accuracy in M_z for phase error in M_{xy} . One use for this pulse sequence is the measurement of the RF field strength, since the actual flip angle θ_a is approximately the negative of magnetization phase $-\angle M_{xy}$. How far can θ_a vary from $\pi/2$ before the error in this approximation is 5° ?

Hint: For these problems the half and double angle formulas are useful

$$\begin{aligned}\sin \theta &= 2 \sin(\theta/2) \cos(\theta/2) \\ \cos \theta &= \cos^2(\theta/2) - \sin^2(\theta/2)\end{aligned}$$

Your answers shouldn't have any half angles.

3. Inverse SLR Transform On the web site are several m-files that implement the inverse SLR transform. These are

b2a.m, mag2mp.m, ab2rf.m

The first is b2a.m. This takes a beta polynomial and returns the consistent, minimum phase, alpha polynomial. It uses mag2mp.m to go from the magnitude profile of alpha to the minimum phase alpha. Then ab2rf.m takes the alpha and beta polynomials, and returns the corresponding RF pulse. There is also an m-file,

ab2inv.m

that takes the output of the simulator `abrm.m`, and returns the M_z profile that is produced.

Given a suitably scaled beta polynomial (i.e. the passband amplitude is $\sin(\theta/2)$), then the RF pulse can be computed as

```
>> a = b2a(b);
>> RF = ab2rf(a,b);
```

One way to think about the inverse SLR transform is that it corrects for the nonlinearity of the Bloch Equation. We can design the beta polynomial using Fourier arguments, just as a small tip angle pulse. Then scale the beta polynomial to $\sin(\theta/2)$, and apply the inverse SLR transform to produce the RF pulse with that beta profile.

a) Design a TBW=8 windowed sinc RF pulse, and scale it to a π radian rotation. Plot the RF pulse scaled to Gauss. Assume a pulse length of 8 ms, and a gradient strength of 0.425 G/cm. Simulate and plot the inversion profile. Choose an interesting range of spatial locations, and compute the inversion profile with

```
>> mz = ab2inv(abrm(rf,x));
```

b) Now, use the windowed sinc waveform as the beta polynomial. First, scale it to the proper value for an inversion. Then find the corresponding minimum phase (and minimum power) alpha polynomial using `b2a.m`. Next compute the RF pulse using `ab2rf.m`. Plot the SLR inversion pulse and the windowed sinc inversion pulse from part (a), both scaled to Gauss. By what factor has the peak amplitude increased?

c) Simulate the inversion profile of the SLR inversion pulse from part (b), and plot it along with the inversion profile of the windowed sinc pulse of part (a).