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ENERGY 281
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Supplemental Notes: Fundamental Theorem of Calculus

In Problem 3 of Homework 1, you use a pseudo-pressure

$$m(p) = \int_{p_{ref}}^{p} \frac{p}{\mu Z}\, dp.$$

The problem requires differentiating m with respect to r and t. This requires using Part 1 of the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\left[\int_a^x f(s)\, ds\right] = f(x),$$

where a is a constant. In words, this theorem states that the derivative of an integral with respect to its upper limit is the integrand, evaluated at that upper limit.

It should be noted that the notation used in the definition of $m(p)$ is somewhat misleading, but unfortunately it is so common (it is Rosalind Archers original notation) that I elected not to change it. Technically, one should *never* use the same letter for the variable of integration and the limit. The variable of integration should only appear *inside* the integral, and nowhere else. With this in mind, a more appropriate definition would be

$$m(p) = \int_{p_{ref}}^{p} \frac{s}{\mu Z}\, ds.$$

Then, by Part 1 of the Fundamental Theorem of Calculus,

$$m'(p) = \left.\frac{s}{\mu Z}\right|_{s=p},$$

the integrand evaluated at the upper limit.

FYI, Part 2 of the Fundamental Theorem of Calculus is the more familiar part,

$$\int_a^b f(x)\, dx = F(b) - F(a),$$

where F is an antiderivative of f; that is, $F'(x) = f(x)$.