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ENERGY 281
Spring Quarter 2007-08
Homework Assignment 1

This assignment is due on Thursday, April 10.

1. The following equation governs the vorticity vector in an incompressible viscous fluid with no body forces, but dont worry if that doesnt mean anything to you. Your task is to write the equation in dimensionless form.
$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{p}{\rho}\right) + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial x^2} \tag{1}$$
where v = velocity, $\frac{m}{s}$
t = time, s
x = distance, m
p = pressure, $\frac{kg}{ms^2}$
ρ = density, $\frac{kg}{m^3}$
μ = viscosity, $\frac{kg}{ms}$
Feel free to introduce any other quantities you may need. There may be several possible solutions to this problem.

2. Instead of a slightly compressible fluid consider the flow of a real gas in a homogeneous medium. The PVT behavior for this case is given by:
$$pV = ZRT \tag{2}$$
Derive the pressure equation in radial coordinates (only consider r and t) for this case. Note that the viscosity μ is now a function of pressure. Hints: Youll need an expression for the gas density.
$$\rho = \frac{pM}{ZRT} \tag{3}$$
Begin with a material balance, derived in the same way as in the 1-D pressure equation at the beginning of Chapter 1 in Archer's notes, except that this is radial flow, so youll be using a cylindrical control volume of height h, radius r, and thickness Δr.

Also, use the gas compressibility
$$c_g = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T.$$

3. Show that by defining a pseudo-pressure:

$$m(p) = \int_{p_{ref}}^{p} \frac{p}{\mu Z}\, dp$$

where p_{ref} is a some reference pressure, the flow equation for a real gas (derived in problem 2) can be simplified considerably.

4. Let K be a positive constant. Use separation of variables, and Fourier sine and cosine series, to solve the PDE

$$\frac{\partial p}{\partial t} = K\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right), \tag{4}$$

on the rectangle $0 < x < 2$, $0 < y < 1$, with initial condition

$$p(x, y, 0) = 3\cos \pi x \sin 2\pi y - 4\cos 2\pi x \sin 4\pi y, \tag{5}$$

and boundary conditions

$$\frac{\partial p}{\partial x}(0, y, t) = \frac{\partial p}{\partial x}(2, y, t) = 0, \quad p(x, 0, t) = p(x, 1, t) = 0. \tag{6}$$