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ENERGY 281
Spring Quarter 2007-08
Homework Assignment 3

This assignment is due on Thursday, May 15. You *must* show your work to receive full credit.

1. Plot the exponential integral solution

$$p_D(r_D, t_D) = \frac{1}{2} Ei\left(-\frac{r_D^2}{4t_D}\right),$$

and the late time approximation to this solution,

$$p_D(r_D, t_D) = -\frac{1}{2}\left(\ln \frac{t_D}{r_D^2} + 0.80907\right),$$

on the same axes for a range of t_D values and $r_D = 1$. It is recommended that you use values of t_D that vary over several orders of magnitude.

For what values of t_D does the late time approximation closely approximate the exponential integral solution? What real time values does this correspond to?

In field units, t_D is defined as

$$t_D = \frac{0.000264kt}{\phi \mu c_t r_w^2},$$

where t is in hours. Choose

$$\begin{aligned} k &= 200 \text{ md}, \\ \phi &= 0.3, \\ \mu &= 1.0 \text{ cp}, \\ c_t &= 10^{-6} \text{ psi}^{-1}, \\ r_w &= 0.3 \text{ ft}. \end{aligned}$$

The quickest, easiest way to do this is in a tool such as MATLAB, Mathematica or Maple.

2. Consider a linear one-dimensional reservoir. The inner boundary condition is constant pressure (p_w) with skin (S). The outer boundary condition is infinite acting. The initial-boundary value problem to be solved is

$$\frac{\partial^2 p}{\partial x^2} = \frac{\phi \mu c_t}{k} \frac{\partial p}{\partial t},$$

with initial condition

$$p(x, 0) = p_i,$$

inner boundary condition

$$p(0, t) - S \left. \frac{\partial p}{\partial x} \right|_{x=0} = p_w,$$

and condition at infinity

$$\lim_{x \to \infty} p(x, t) = p_i.$$

(a) First, write the problem in dimensionless form. Use the same transformations for x and t as in the Lecture 1 notes. For p, use the same transformation as in the case of a constant pressure boundary condition. Non-dimensionalize the skin using the transformation $S_D = S/L$, where L is the length scale of the reservoir.

(b) Next, solve the dimensionless differential equation in Laplace space.

(c) Finally, invert the transform. Use either a table, or the Maple function `invlaplace`. If you use a table, indicate which table. If you use Maple, submit your code and output.

3. Solve the Cauchy problem

$$\frac{\partial^2 T}{\partial x^2} = \alpha \frac{\partial T}{\partial t}, \quad -\infty < x < \infty, \quad t > 0,$$

where α is a constant, with initial condition

$$T(x, 0) = e^{-\pi x^2}, \quad -\infty < x < \infty.$$

Use *both* the Fourier transform and Laplace transform *together* to obtain an *algebraic* equation that can easily be solved, and then invert both transforms to obtain $T(x, t)$. Be careful with your notation to keep track of your transforms.

4. Consider the following sampled signal $g(t)$ on the interval $[0, 1)$:

$$g(t) = \begin{cases} 1 & 0 \leq t < 0.25 \\ 0.5 & 0.25 \leq t < 0.5 \\ 0.75 & 0.5 \leq t < 0.75 \\ 0.25 & 0.75 \leq t < 1 \end{cases}.$$

This signal is shown in Figure 1.

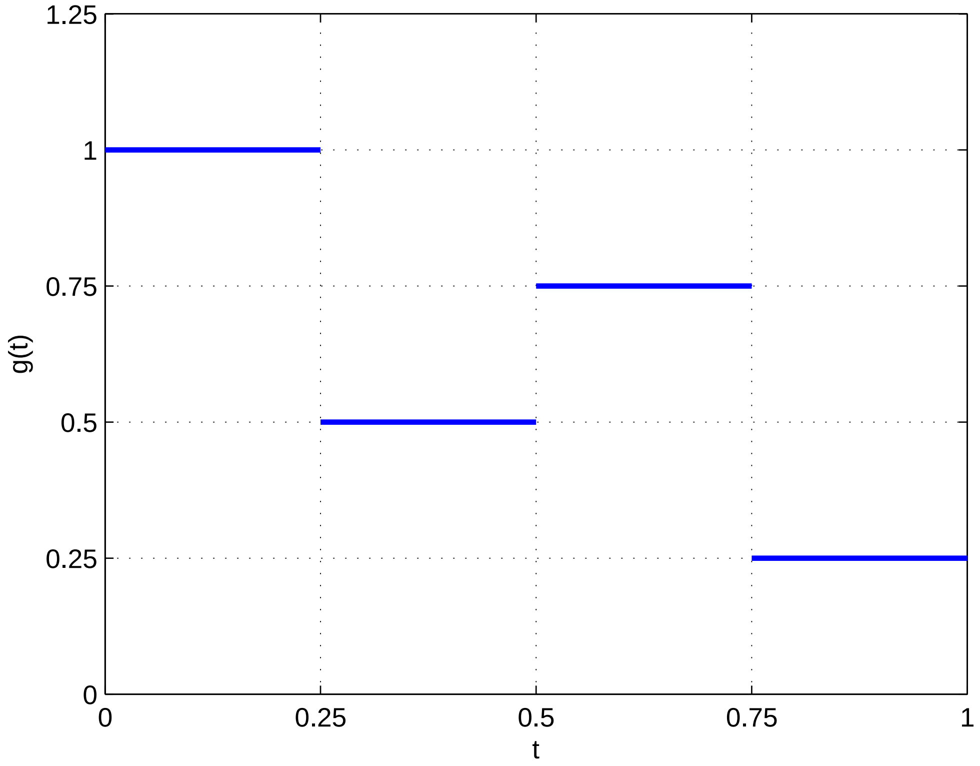

Figure 1: Signal for Problem 4

(a) This signal can be expressed as a sum of Haar smoothing functions at a particular scale M,

$$g(t) = \sum_{n=-\infty}^{\infty} \phi_{M,n}(t) s_{M,n}.$$

provided that this scale is sufficiently fine. What is the largest value of M for which this is possible? In other words, what is the largest M such that $g \in V_M$? Recall that larger values of M correspond to coarser scales.

(b) Perform the Haar decomposition of g to compute the detail coefficients $\{d_{M+1,n}\}$, $\{d_{M+2,n}\}$, and $\{s_{M+2,n}\}$, where M is the scale determined in part (a).

(c) Compress the signal by setting to zero the detail coefficients which satisfy

$$|d_{m,n}| \leq 0.5.$$

Reconstruct the compressed signal using the relation

$$\sum_{n=-\infty}^{\infty} s_{m,n}\phi_{m,n}(t) = \sum_{n=-\infty}^{\infty} s_{m+1,n}\phi_{m+1,n}(t) + \sum_{n=-\infty}^{\infty} d_{m+1,n}\psi_{m+1,n}(t).$$

(d) If $\tilde{g}$ is a compression of a signal g, then the compression ratio is given by

$$100\frac{\|\tilde{g}\|_{L_2}^2}{\|g\|_{L_2}^2} = 100\frac{\sum_{m,n=-\infty}^{\infty} \tilde{d}_{m,n}^2}{\sum_{m,n=-\infty}^{\infty} d_{m,n}^2},$$

where $\{\tilde{d}_{m,n}\}$ is the set of detail coefficients for the compressed signal. Compute and interpret the compression ratio for the compression performed in part (c). *Hint:* $\tilde{d}_{M+3,0} = d_{M+3,0} = \sqrt{2}s_{M+2,0}$.