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ENERGY 281
Spring Quarter 2007-08
Homework Assignment 4

This assignment is due on Thursday, May 22. You *must* show your work to receive full credit.

1. Consider the boundary value problem given by the one dimensional advection-diffusion-reaction equation

$$\frac{d}{dx}\left(u - \frac{du}{dx}\right) = 0, \quad x \in [0,1],$$

with the boundary conditions

$$u(0) - \left.\frac{du}{dx}\right|_{x=0} = 1, \quad u(1) = 1.5.$$

   (a) Obtain the weak form of the equation so that the space of trial and test functions are the same.

   (b) Consider a discretization of the domain in which the interval $[0,1]$ is divided up into N finite elements, each with length h. Using M generic basis functions $\phi_1, \phi_2, \ldots, \phi_M$, where the value of M depends on N and the boundary conditions, write down the Galerkin finite element equations, in terms of inner products of the basis functions and their derivatives.

   (c) Assume the basis functions are the piecewise linear "hat" functions used in the notes for lectures 11 and 12. Write down the elements of the stiffness matrix K and the load vector F, not accounting for the boundary conditions.

   (d) Incorporate the boundary conditions into the stiffness matrix and load vector.

   (e) For $N = 4$, 8 and 16, solve the system of equations from part 1d, using a tool such as MATLAB. Plot the approximate solution against the analytical solution (include all four solutions on the same plot). How good is the approximation? How does the accuracy improve as N increases?