Document Analysis

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Outline

Word count vectors

- fix a dictionary containing n different words
- associate *word count vector* a with a document
- $a_i =$ number of times word i appears in the document
- $h = a/\mathbf{1}^T a$ is *word frequency* or *histogram* vector
- other normalizations and representations (*e.g.*, tf-idf, bi-grams) are sometimes used

Pre-processing

documents are *pre-processed* before words are counted

- stemming: remove endings from words
  - cat, cats, catty $\rightarrow$ cat
  - stemmer, stemmed, stemming $\rightarrow$ stem
- filter (remove) 'stop' words
  - short words: the, is, at
  - most common words: what, this, how
  - extremely uncommon words

Example

Dictionary	Doc A	Doc B	Doc C
bankrupt	0	3	0
baseball	0	0	3
bat	3	0	3
harry	3	0	0
homerun	0	1	2
legendary	4	1	1
magic	3	0	1
sport	0	0	4
stock	0	3	0
⋮	⋮	⋮	⋮

can you guess the topics of each document?

Document-term matrix

- we have a *corpus* (collection) of N documents, with word count n-vectors $a_1, \ldots, a_N$
- *document-term* matrix is $N \times n$ matrix A, with $A_{ij} =$ number of times word j appears in document i
- rows of A are $a_1^T, \ldots, a_N^T$
- jth column of A shows occurences of word j across corpus

Outline

Similarity measures

- two documents, with word count vectors a_1, a_2, histogram vectors h_1, h_2
- *distance measure* (of dissimilarity): $\|h_1 - h_2\|$
- *angle measure* (of dissimilarity): $\angle(a_1, a_2) = \angle(h_1, h_2)$
- we expect these to be small when the documents have the same topics, genre, or author, and larger otherwise

Example

- 4 chapters with histograms h_1, h_2, h_3, h_4
- dictionary is 1000 most common words

Harry Potter 1. Harry did the best he could, trying to ignore the stabbing pains in his forehead, which had been bothering him ever since his trip into the forest . . .

Harry Potter 2. "Severus?" Quirrell laughed, and it wasn't his usual quivering treble, either, but cold and sharp . . .

Foundations 1. Gaal Dornick, using nonmathematical concepts, has defined psychohistory to be that branch of mathematics which deals with the reactions of human conglomerates to fixed social and economic stimuli . . .

Foundations 2. The trial (Gaal supposed it to be one, though it bore little resemblance legalistically to the elaborate trial techniques Gaal had read of) had not lasted long . . .

Example

- $\|h_i - h_j\|$ $(\times 100)$

	HP1	HP2	FO1	FO2
HP1	0	0.4	1.4	1.3
HP2		0	1.4	1.2
FO1			0	0.8
FO2				0

- $\angle(h_i, h_j)$ (in degrees)

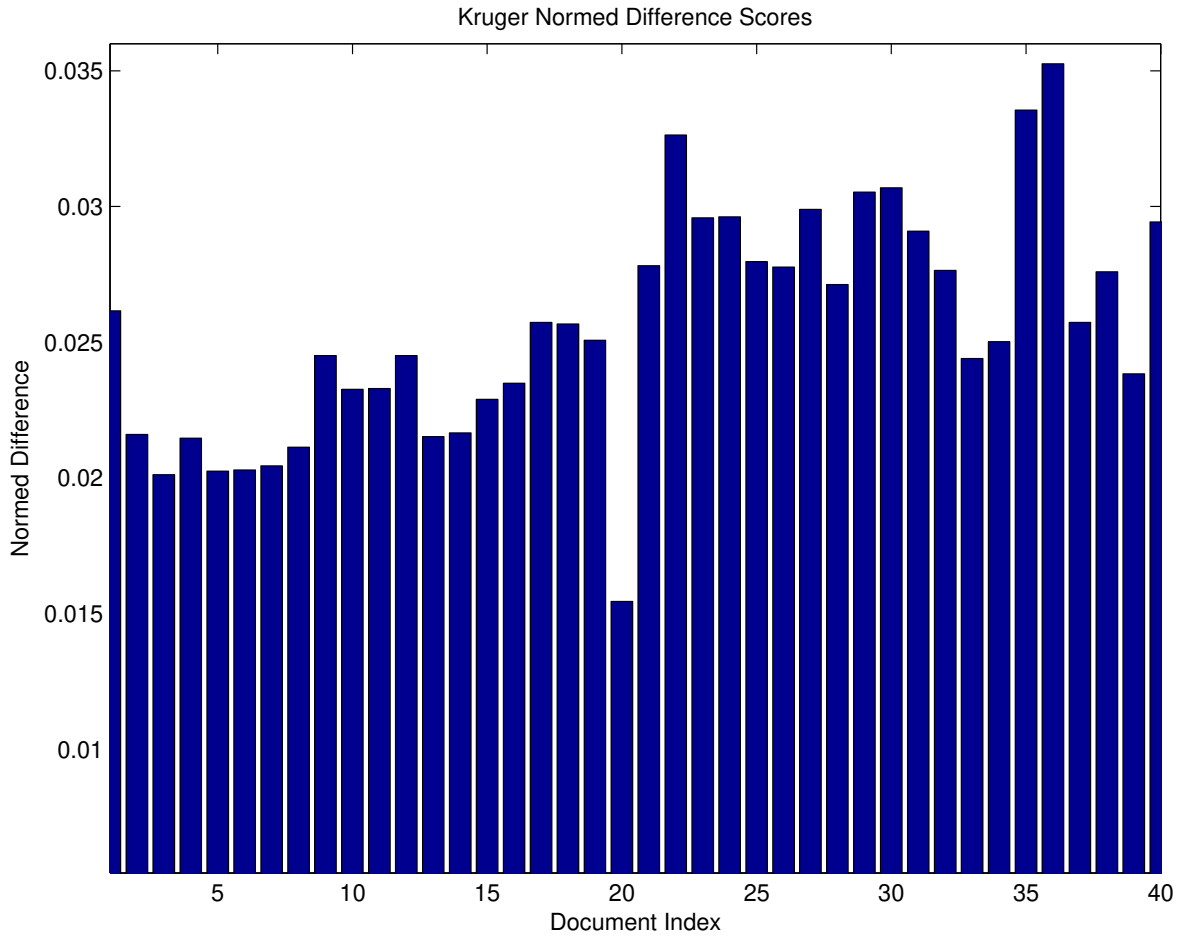
	HP1	HP2	FO1	FO2
HP1	0	40.8	84.7	84.1
HP2		0	84.1	82.0
FO1			0	74.1
FO2				0

Example

- 40 documents, with word count histograms $h_1, \ldots, h_{40}$
  - 1–20 are news articles
  - 20 is by Paul Krugman
  - 21–40 are Harry Potter excerpts
- another article by Paul Krugman, with histogram b
- dictionary capped at 1000 words
- let's look at $\angle(h_i, b)$ and $\|h_i - b\|$, $i = 1, \ldots, 40$

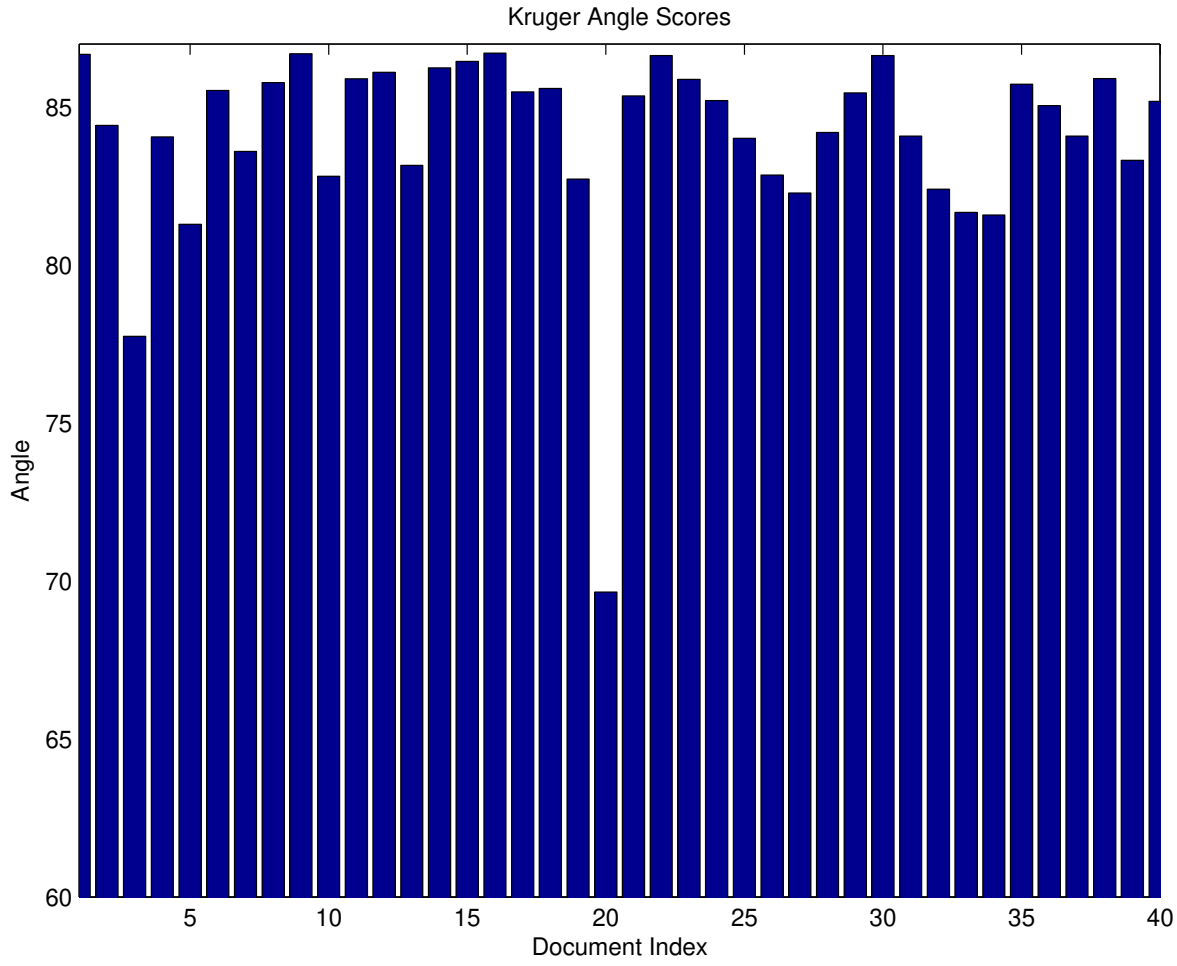
Example

$$\|h_i - b\|,\ i = 1, \ldots, 40$$



Example

$\angle(h_i, b),\ i = 1, \ldots, 40$



Outline

k-means on histogram vectors

- start with corpus of N documents with histograms $h_1, \ldots, h_N$
- use k-means algorithm to cluster into k groups of documents
- groups usually have similar topics, genre, or author
- this is sometimes called (automatic) *topic discovery*
- centroids $z_1, \ldots, z_k$ are also histograms

Example

- corpus of 555 documents, dictionary capped at 1000 most common words
  - 185 Harry Potter excerpts
  - 185 education articles
  - 185 sports articles
- use k-means with $k = 3$; best of 10 random initializations
- results:

Cluster	Sports	Education	Harry Potter
1	183	39	19
2	2	146	0
3	0	0	166

Example

words associated largest coefficients of centroid vectors:

Cluster 1	player	year	league	football	team
Cluster 2	student	education	school	university	college
Cluster 3	harry	hermione	ron	eye	said

Example

- let's use our three cluster centroids to classify *new* documents:
  - 15 Harry Potter excerpts
  - 15 education articles
  - 15 sports articles

- results (in a *confusion matrix* or table):

predicted ↓ true →	Sports	Education	Harry Potter
Sports	15	0	0
Education	0	15	0
Harry Potter	0	0	15

Outline

Document count vectors

- we have a corpus of N documents
- associate with a word its *document count vector* b
- $b_i =$ number of times word appears in document i
- b_i are columns of document-term matrix A
  (word count vectors are rows of A)
- normalized document count (histogram) vector is $g = b/\mathbf{1}^T b$
- words that appear in similar ways across the corpus have close document count or histogram vectors

Word clustering

- use k-means algorithm on histograms g_i to partition words into k groups
- words in same cluster tend to co-appear in the same documents in the corpus

Example

- same example as above (555 documents, 1000 words)
- run k-means word clustering with $k = 50$
- words from some of the clusters:

investigate	charge	lawsuit	allege	title
concuss	injury	draft	retire	brain
gryffindor	firebolt	slytherin	broom	penalty
world	team	soccer	game	brazil

Outline

Regression model

- goal: predict a number y (*e.g.*, grade, score, rating) from a document's word count vector a
- regression model:

$$\hat{y} = w^T a + v$$

  - $\hat{y}$ is predicted value of y
  - a is a document word count vector
  - w is weight vector; v is offset
  - we are to choose w and v so $y \approx \hat{y}$
- we have a *training set* of N documents and their ('true') y values

$$(a_1, y_1), \ldots, (a_N, y_N)$$

Regression

- want w, v for which $y_i \approx \hat{y}_i = w^T a_i + v$
- we'll judge regression prediction error via its RMS value

$$\left(\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \right)^{1/2}$$

- choose w, v to minimize

$$\sum_{i=1}^{N} (\hat{y}_i - y_i)^2 + \lambda \|w\|^2 = \|Aw + v\mathbf{1} - y\|^2 + \lambda \|w\|^2$$

  - $\lambda > 0$ is regularization parameter
  - first term is RMS prediction error (squared, times N)

Validation

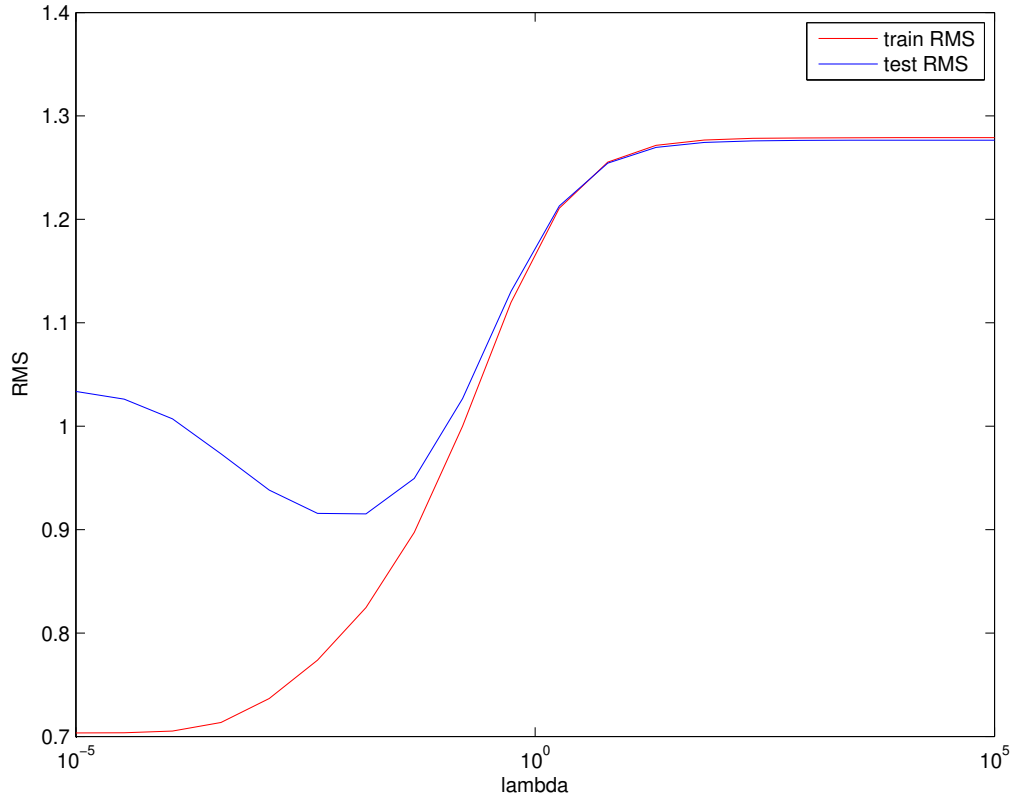
- we are interested on w, v that give good predictions on *new, unseen* documents
- so we *test* or *validate* w, v on a different set of documents, the *test set*
- we choose λ so that the RMS prediction error *on test set* is small

Example

- set of 8884 Yelp reviews with at least 50 words
- dictionary is 1000 most common words, *e.g.*, place, great, food, good
- reviews y_i have values in $\{1, 2, 3, 4, 5\}$
  - $\mathbf{avg}(y) = 3.56$, $\mathbf{std}(y) = 1.28$
  - so always guessing $\hat{y} = 3.56$ gives RMS error 1.28
- divide documents into training set (6218) and test set (2666)

Example

RMS error versus λ



using $\lambda = 150$ gives RMS test error ≈ 0.92

Example

weights with largest values

word	weight
perfect	0.196
best	0.178
five	0.164
fantastic	0.160
amazing	0.158
awesome	0.157
⋮	⋮
terrible	-0.231
rude	-0.280
horrible	-0.281
worst	-0.284
bland	-0.298

Example

- now let's take prediction $\hat{y}$ and round it to $\{1, 2, 3, 4, 5\}$
- results:

Prediction error	Train	Test	Predicting 4
perfect	47%	41%	33.3%
off by one	47%	50%	44%
off by two	5.6%	8.9%	12%
off by three	0.20%	0.53%	9.7%
off by four	0%	0%	0%

Example

confusion matrix on training set

predicted ↓ / true →	1	2	3	4	5
1	95	308	183	10	0
2	34	261	441	44	0
3	2	94	594	327	10
4	0	10	403	1392	250
5	0	2	104	1072	582

Example

confusion matrix on test set

predicted ↓ true →	1	2	3	4	5
1	37	108	111	10	0
2	14	89	202	35	0
3	2	46	224	162	4
4	0	7	224	554	135
5	0	4	76	429	193

Outline

Document classification

- documents have *labels* from a finite set, *e.g.*,
  - *email* or *spam*
  - *excerpt from Harry Potter* or *not*
  - *about sports* or *news* or *neither*
- divides documents into *classes*
- we'll focus on binary case, with two labels
- *document classification:* given word count vector a, guess which class the document is in
- judge classification performance by error rate on test set

Least squares classification

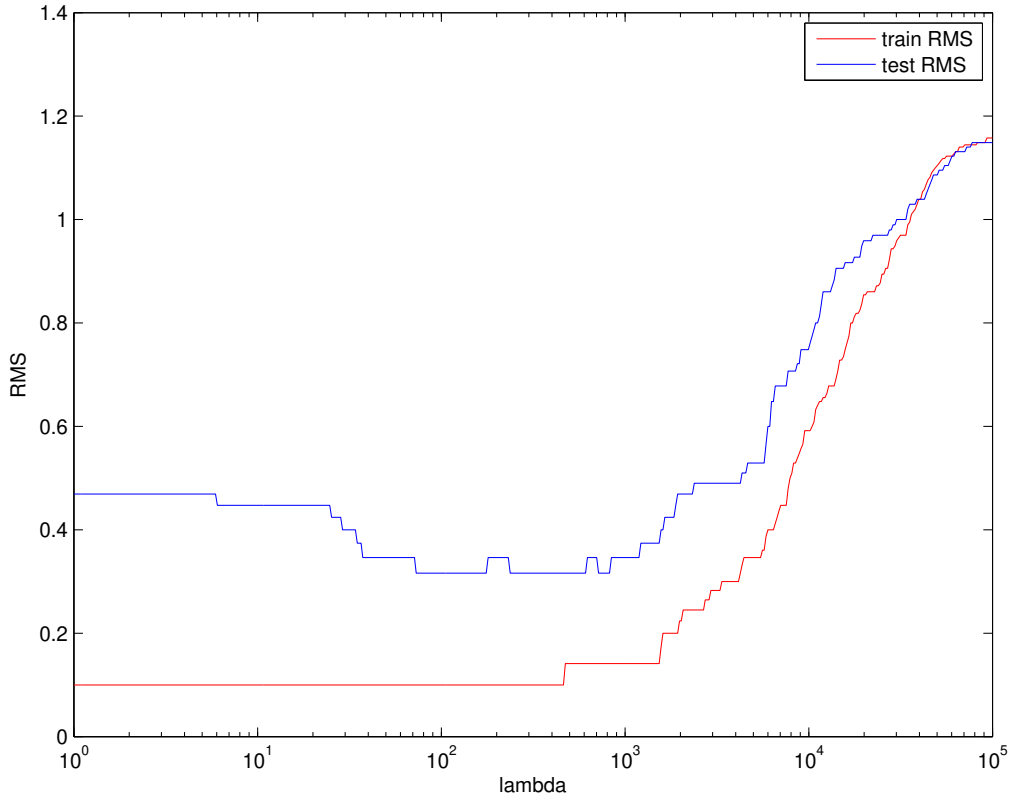
- we use label $y_i = 1$ for one class and $y_i = -1$ for the other
- find regression model $\tilde{y} = w^T a + v$
- guess (classify) document using $\hat{y} = \mathbf{sign}(\tilde{y})$
- choose regularization parameter λ by error rate on test set

Example

- same corpus of 555 documents: sports, education, and Harry Potter
- split into training set (370 documents) and test set (185 documents)
- predict *sports* articles versus *not sports*
- label sports articles with 1, others -1

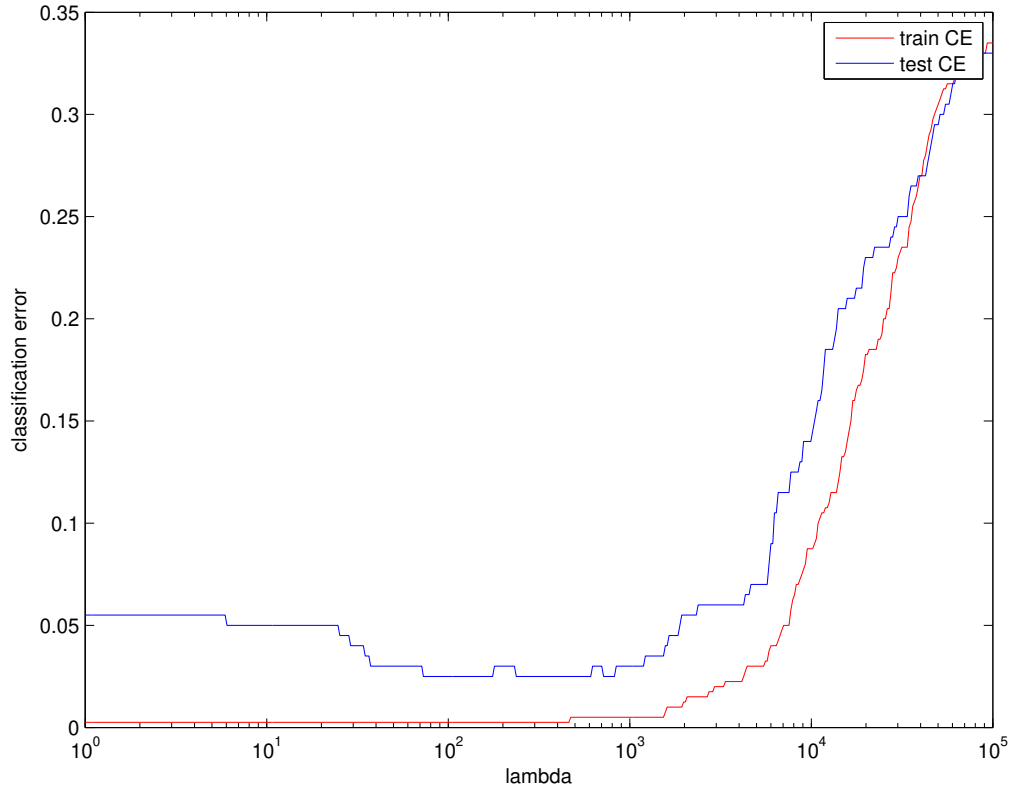
Example

RMS prediction error versus λ



Example

classification error versus λ



choosing $\lambda = 285$ gives test set error rate around 2%

Example

weights with largest values

word	weight
olympics	0.0532
play	0.0491
football	0.0464
player	0.0402
final	0.0359
committee	0.0341
⋮	⋮
school	-0.0230
get	-0.0249
read	-0.0269
campus	-0.0320
harry	-0.0360