# ENGR 76
# Information Science and Engineering

## Lecture 11: Digital Communication - On-Off Keying

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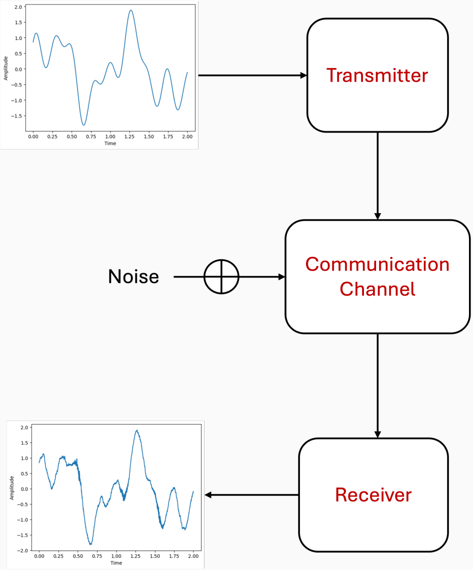
# Communication

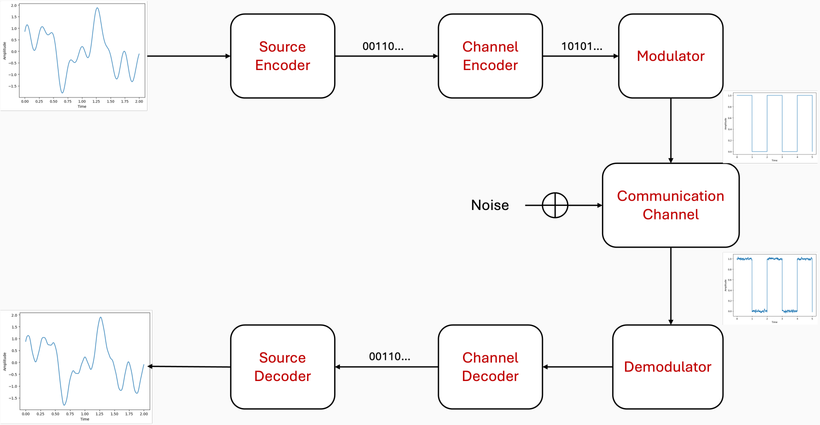
# Communication

- Sending information reliably over noisy channels

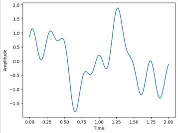
# Analog Communication



# Digital Communication



# Source Coding




x(t)

Sampling — Lossless (under Nyquist condition)

$x[n] \in \mathbb{R}$

Quantization — Lossy

X_1, X_2, X_3... $\in \mathcal{X}$

Coding — Lossless. Example: Huffman Coding

Encoded Bit Sequence
e.g., 0101011001100

# Channel Coding

- Adding redundancy (extra bits) so that errors caused by the channel can be detected and corrected at the receiver
- Error correction codes
- **Next week**

# Modulation

- Cannot directly send bits through an electric cable
- Mapping bits to a continuous-time signal
- **This week**

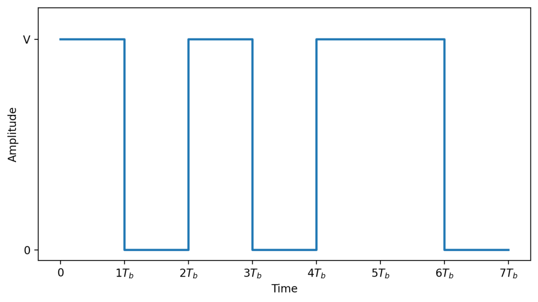
# On-Off Keying: Modulation and Demodulation

# On-Off Keying

- Consider the bit sequence: $1010110$
- Simplest continuous-time waveform constructed from this bit sequence?

# On-Off Keying

- Consider the bit sequence: $1010110$
- **On-Off Keying**




- **Bit Duration** or **Symbol Duration**: $T_b$
- Signal Amplitude: $V$

# On-Off Keying

- Let $\{b_m\}$ denote the bit sequence for $m = 1, 2, \ldots$
- Fix bit duration $T_b$ and amplitude $V$:

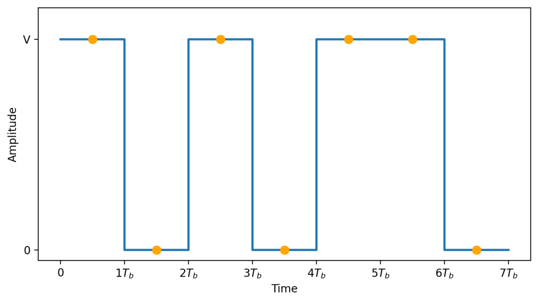
$$x(t) = \begin{cases} V, & t \in [(m-1)T_b, mT_b) \text{ with } b_m = 1, \\ 0, & t \in [(m-1)T_b, mT_b) \text{ with } b_m = 0, \end{cases} \qquad \text{for each } m \geq 1.$$

# Demodulation

- Transmitted $x(t)$ through the noisy channel and received $y(t)$
- Mapping the received continuous-time waveform to sequence of bits
- Ideas?

# Idea I: Sampling

- Sample at midpoint of each bit duration
  - $\hat{b}_m = x\left((m-1)T_b + \frac{T_b}{2}\right)$
- Suppose noiseless, i.e., $y(t) = x(t)$
  - Works perfectly if channel is noiseless



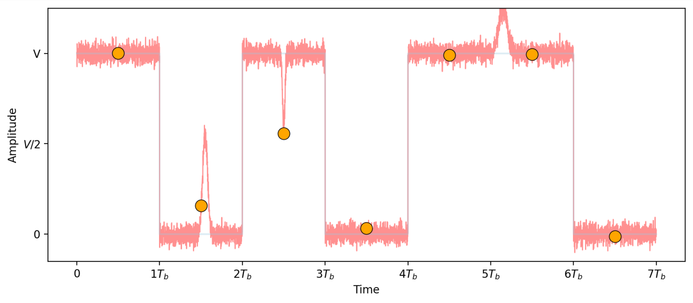

- But what if channel is noisy?

# Idea I: Sampling

- Sample at midpoint of each bit duration
- If sample is larger than $V/2$, then output $\hat{b}_m = 1$
- If sample is smaller than $V/2$, then output $\hat{b}_m = 0$

# Idea I: Sampling

- Sampling at a single time instant is susceptible to noise
  - Especially in presence of bursty noise




- For example, in this case $\hat{b}_3 = 0$ even though $b_3 = 1$
  - Even though most of the signal has little noise

# Power Detection and Thresholding

1. Divide time into bit intervals of length $T_b$, i.e.,

$$[(m-1)T_b, \, mT_b], \qquad m = 1, 2, \ldots$$

2. For each bit interval, compute the average received power

$$p_m \;=\; \frac{1}{T_b} \int_{(m-1)T_b}^{mT_b} y^2(t) \, dt.$$

3. Compare the power to a threshold and decide the bit:

$$\hat{b}_m = \begin{cases} 1, & p_m > p_{\text{thresh}}, \\ 0, & p_m \le p_{\text{thresh}}. \end{cases}$$

# Power Detection and Thresholding

- Suppose noiseless:
  - $y(t) = V$ for $t \in [(m-1)T_b, mT_b]$ if $b_m = 1$
  - $y(t) = 0$ for $t \in [(m-1)T_b, mT_b]$ if $b_m = 0$
- What value to choose for $p_{thresh}$?

# Power Detection and Thresholding

**Noiseless Case:**

- When $b_m = 0$,

$$p_m = \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} y^2(t)\,dt = \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} 0\,dt = 0$$

- When $b_m = 1$,

$$\begin{aligned} p_m &= \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} y^2(t)\,dt = \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} V^2\,dt \\ &= \frac{1}{T_b}\big(V^2 m T_b - V^2(m-1)T_b\big) \\ &= V^2 \end{aligned}$$

# Power Detection and Thresholding

**Noiseless Case:** We want to choose $p_{thresh}$ such that:

- the received power $(V^2)$ is larger than $p_{thresh}$ when $b_m = 1$
  - will ensure $\hat{b}_m = 1$
- the received power $(0)$ is smaller than $p_{thresh}$ when $b_m = 0$
  - will ensure $\hat{b}_m = 0$

**Answer:** $p_{thresh}$ can be chosen to be any value between $0$ and $V^2$

- Will ensure error-free decoding

# Power Detection and Thresholding

**Noisy Case:**

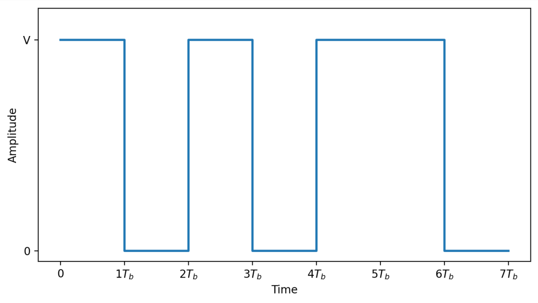
- Same strategy and calculation for $p_m$
- Value of threshold $p_{thresh}$ depends on the noise properties
- Typically more robust to noise than sampling at a single time instant
    - Cannot guarantee error-free decoding in general

# Properties of OOK Modulation

# Design Parameters

- Amplitude $V$
  - A higher $V$ is better - improves signal-to-noise ratio
  - Lower probability of bit error
  - Restricted by power constraints, hardware effects, etc.
- Bit duration $T_b$
  - How to choose $T_b$?

# Bit Rate




- One bit of information is transmitted in $T_b$ seconds

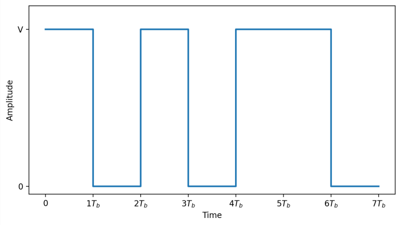
$$\boxed{\text{Bit Rate } R_b = \frac{1}{T_b} \text{ bits/sec}}$$

- A higher bit rate is better
- Decreasing $T_b$ increases the bit rate
- What stops us from making $T_b$ smaller and smaller?

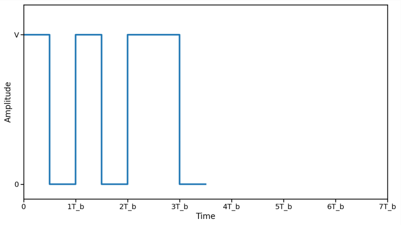
# Smaller $T_b$

- Smaller $T_b$ can lead to a higher probability of bit error
  - Power detection averages noise over a smaller interval
  - Error correcting codes can mitigate this to an extent
- More fundamental limitation: **Bandwidth**

# Bandwidth of OOK Modulation



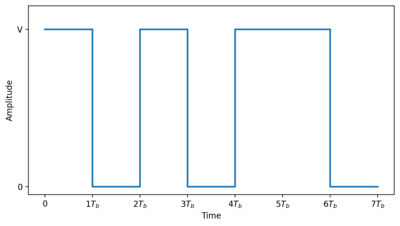

**(a)** $x_1(t)$ with bit rate $1/T_b$



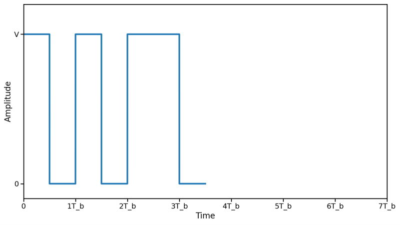

**(b)** $x_2(t)$ with bit rate $2/T_b$

- Let $x_1(t)$ be the waveform with bit rate $1/T_b$
- Let $x_2(t)$ be the waveform with bit rate $2/T_b$
- How are they related?

# Bandwidth of OOK Modulation




**(a)** $x_1(t)$ with bit rate $1/T_b$




**(b)** $x_2(t)$ with bit rate $2/T_b$

- $x_2(t) = x_1(2t)$
  - If the spectrum of $x_1(t)$ lies in $[0, B]$, then what about $x_2(t)$?

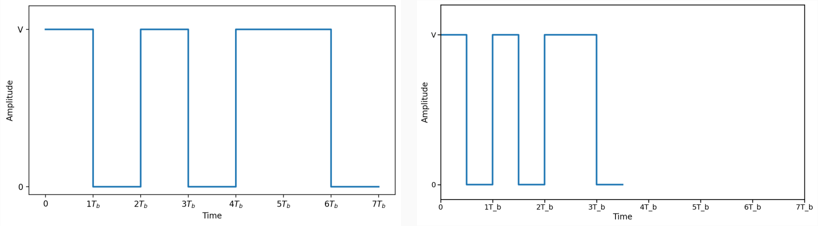
# Example

- Let $z(t) = 1 + 3\cos(2\pi 5t)$
- Frequency band of $z(t)$?
- What about $z(2t)$?

# Example

- Let $z(t) = 1 + 3\cos(2\pi 5t)$
- Frequency band of $z(t)$ is $[0, 5]$ Hz
- $z(2t) = 1 + 3\cos(2\pi 10t)$
- Frequency band of $z(2t)$ is $[0, 10]$ Hz

# Bandwidth of OOK Modulation




**(a)** $x_1(t)$ with bit rate $1/T_b$

**(b)** $x_2(t)$ with bit rate $2/T_b$

- $x_2(t) = x_1(2t)$
  - If the spectrum of $x_1(t)$ lies in $[0, B]$, then the spectrum of $x_2(t)$ lies in $[0, 2B]$
  - The bandwidth is doubled as we double the bit rate!

# Why does bandwidth matter?

**Reason I:** Channels are frequency selective

- Physical channels have limits
- Different frequency components experience different attenuation and distortion
- We want to shape the spectrum of the transmitted signal such that it matches the frequency band where the channel response is favorable

# Why does bandwidth matter?

**Reason II:** Bandwidth is scarce and expensive

- Especially critical in wireless channels
- Electromagnetic spectrum is a limited and valuable resource
  - Regulated by the Federal Communication Commission (FCC)
- Wireless transmitters are allocated non-overlapping frequency bands to avoid interference
  - Next class

# Bandwidth of OOK Modulation

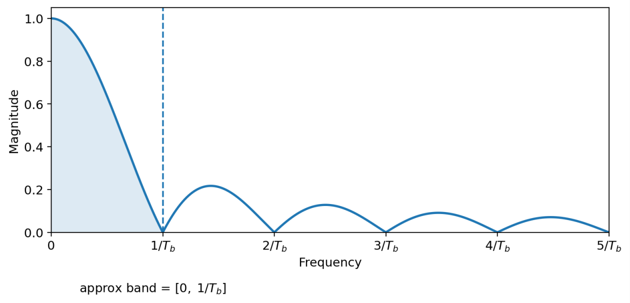
**Fact**

For OOK modulation with bit duration $T_b$ (equivalently, bit rate $1/T_b$), the waveform is *approximately bandlimited* to the frequency range $[0,\ 1/T_b]$.

- Higher bit rate $\iff$ Larger band
- **Baseband** signal with approximate bandwidth $1/T_b$

# Meaning of approximate band

- The exact spectrum is sinc-shaped
  - Zero-crossings at multiples of $1/T_b$
- Only the first lobe considered - beyond which the spectrum is considered small and ignored



# What if we want exactly bandlimited?

- OOK modulation is only approximately bandlimited
- Using sinc-shaped pulses gives exactly bandlimited signals
    - Same intuition as sinc interpolation: a signal is constructed as a weighted sum of shifted sinc functions
- More details and examples provided in lecture notes (optional)

# Thank You!