ENGR 76

Information Science and Engineering

Lecture 12: Modulation and Demodulation

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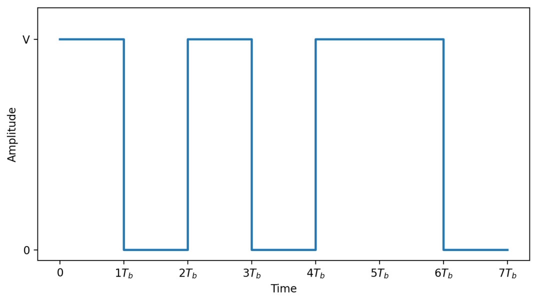
On-Off Keying

Modulation

- Cannot directly send bits through an electric cable
- Mapping bits to a continuous-time signal

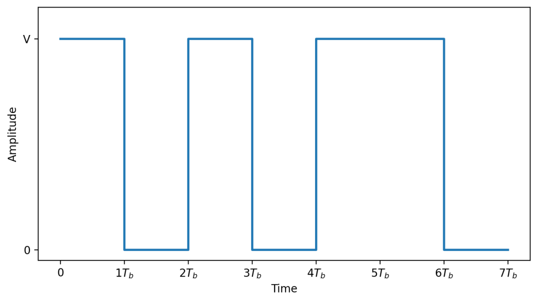
On-Off Keying

- Consider the bit sequence: 1010110
- **On-Off Keying**



- **Bit Duration** or **Symbol Duration**: T_b
- Signal Amplitude: V

Bit Rate



- One bit of information is transmitted in T_b seconds

$$\boxed{\text{Bit Rate } R_b = \frac{1}{T_b} \text{ bits/sec}}$$

Bandwidth of OOK Modulation

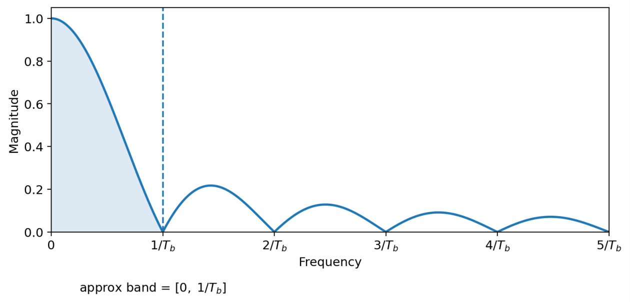
Fact

For OOK modulation with bit duration T_b (equivalently, bit rate $1/T_b$), the waveform is *approximately bandlimited* to the frequency range $[0, 1/T_b]$.

- Higher rate $\iff$ Larger band
- **Baseband** signal with approximate bandwidth $1/T_b$

Meaning of approximate band

- The exact spectrum is sinc-shaped
  - Zero-crossings at multiples of $1/T_b$
- Only the first lobe considered - beyond which the spectrum is considered small and ignored



- What does this plot mean?

Recall: Spectrum of a signal of length L

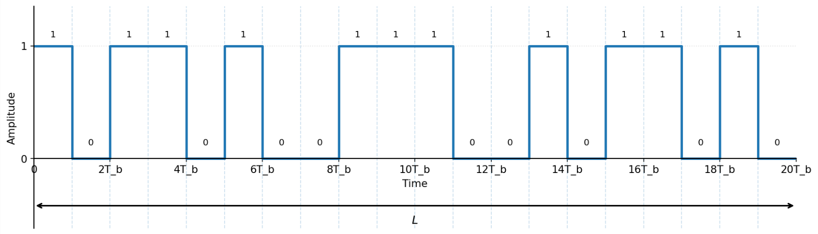
- Consider a continuous-time signal of length (duration) L seconds
- Recall that it can be represented as

$$x(t) = b_0 + \sum_{j=1}^{\infty} a_j \sin\left(2\pi \frac{j}{L} t\right) + b_j \cos\left(2\pi \frac{j}{L} t\right)$$

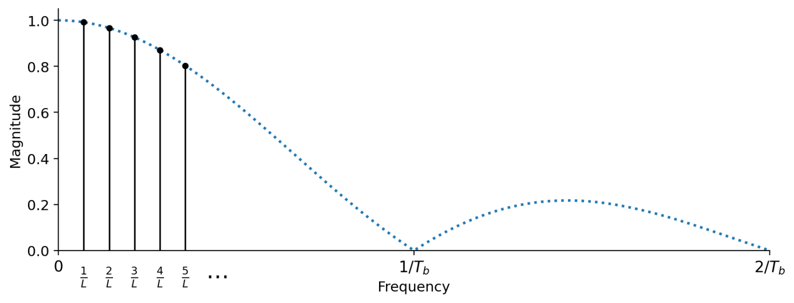
- Recall that the spectrum can be plotted using $A_j = \sqrt{a_j^2 + b_j^2}$
  - Stems at discrete frequencies j/L

Spectrum of OOK Modulation

- Consider $x(t)$ constructed using OOK Modulation
  - Bit Duration: T_b seconds
  - Total length of signal: L seconds
  - Typically $L \gg T_b$



Spectrum of OOK Modulation

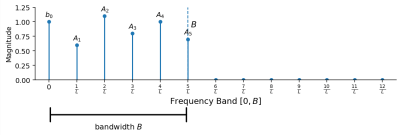


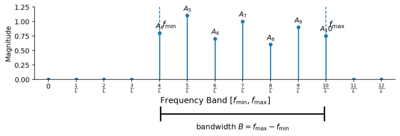
- Black stems - actual coefficient magnitudes A_j
- Blue dotted curve - sinc envelope to illustrate decay of the coefficients
  - Approximate band is just the first lobe $[0, 1/T_b]$ Hz

OOK

- We have a signal which is (approximately) bandlimited to $[0, B]$
  - We know dependence between B and rate $1/T_b$ ($B = 1/T_b$)
- Are we done?

Recall: Baseband vs Passband





- Baseband Signal:
  - Frequency band: $[0, B]$ Hz
  - Bandwidth: B Hz
- Passband Signal:
  - Frequency band: $[f_{min}, f_{max}]$ Hz
  - Bandwidth: $B = f_{max} - f_{min}$ Hz

Issues with Baseband Signal

Reason I: Antenna Size

- We cannot really transmit baseband signals over the air
- Antenna size inversely proportional to frequency
    - Baseband signals would require impractically large antennas

Reason II: Multiple Carriers

- In practice, many users transmit at the same time
- If all signals were baseband, they would overlap
- Each user's signal shifted to a different *carrier frequency*
- Frequency Allocation Chart
- Example: FM broadcast band is 88.0 - 108.0 MHz
  - Divided into 100 channels
  - Each channel occupies 200 kHz
  - FM Channels in California

Baseband to Passband

Example

- Suppose $z_1(t) = 1 + \cos(2\pi 100t)$
- Frequency band of $z_1(t)$?
- $z_2(t) = \sin(2\pi 1000t) \times z_1(t)$
  - $z_2(t) = ?$
  - Frequency band of $z_2(t)$?
  - *Hint:* $\sin(a)\cos(b) = \frac{1}{2}(\sin(a+b) + \sin(a-b))$

Example

- Suppose $z_1(t) = 1 + \cos(2\pi 100t)$
- Frequency band of $z_1(t)$: $[0, 100]$ Hz
- $z_2(t) = \sin(2\pi 1000t) \times z_1(t)$

$$\begin{aligned} z_2(t) &= \sin(2\pi 1000t) \times (1 + \cos(2\pi 100t)) \\ &= \sin(2\pi 1000t) + \sin(2\pi 1000t)\cos(2\pi 100t) \\ &= \sin(2\pi 1000t) + \frac{1}{2}\sin(2\pi 1100t) + \frac{1}{2}\sin(2\pi 900t) \end{aligned}$$

- $z_2(t) = 0.5\sin(2\pi 900t) + \sin(2\pi 1000t) + 0.5\sin(2\pi 1100t)$
- Frequency band of $z_2(t)$: $[900, 1100]$ Hz
  - $[1000 - 100, 1000 + 100]$ Hz

Baseband to Passband

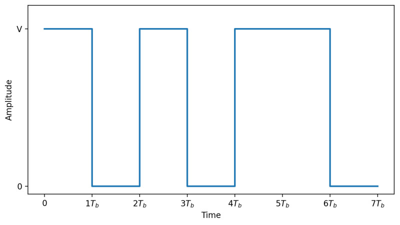
- Suppose $x(t)$ is bandlimited to $[0, B]$ Hz
- If $f_c > B$, then $s(t) = x(t) \times \sin(2\pi f_c t)$ is bandlimited to $[f_c - B, f_c + B]$ Hz

Generating Passband Signal

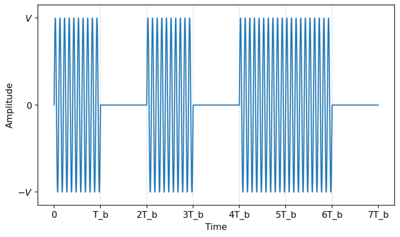
- Consider the bit sequence: 1010110
- Consider $x(t)$ generated using on-off keying with bit duration T_b
- Let $s(t) = x(t) \times \sin(2\pi f_c t)$ with $f_c \gg 1/T_b$

Generating Passband Signal

- Consider the bit sequence: 1010110
- Consider $x(t)$ generated using on-off keying with bit duration T_b
- Let $s(t) = x(t) \times \sin(2\pi f_c t)$



(a) $x(t)$



(b) $s(t)$

Carrier Wave and Carrier Frequency

- $x(t)$ is the message signal
- $\sin(2\pi f_c t)$ 'carries' it at high frequencies: **Carrier Wave**
- f_c: **Carrier Frequency**
  - Typically $f_c \gg 1/T_b$

Bandwidth

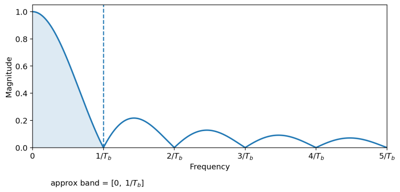
Fact

For OOK modulation with bit duration T_b (equivalently, bit rate $1/T_b$), the baseband waveform $x(t)$ is *approximately* bandlimited to the frequency range $[0,\ 1/T_b]$.

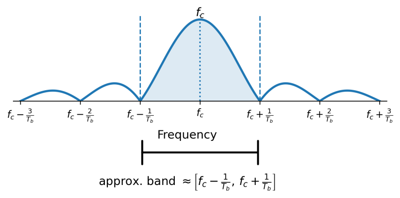
The passband signal constructed using $s(t) = x(t) \times \sin(2\pi f_c t)$, where $f_c > 1/T_b$ is *approximately* bandlimited to the frequency range $[f_c - 1/T_b, f_c + 1/T_b]$.

- Bit Rate R_b is still $1/T_b$

Shifting OOK spectrum to Passband



(a) Spectrum of $x(t)$



(b) Spectrum of $s(t)$

- For the same rate $1/T_b$, bandwidth of baseband signal is $1/T_b$ and of passband signal is $2/T_b$

Constructing appropriate passband signal

- Suppose the band $[f_{min}, f_{max}]$ has been allocated to us for communication
- We want to utilize the complete band
- Want $f_c - 1/T_b = f_{min}$ and $f_c + 1/T_b = f_{max}$
- $f_c = ?$ and $T_b = ?$

Constructing appropriate passband signal

- Suppose the band $[f_{min}, f_{max}]$ has been allocated to us for communication
- We want to utilize the complete band
- Want $f_c - 1/T_b = f_{min}$ and $f_c + 1/T_b = f_{max}$
- **Choose $f_c = \frac{f_{min}+f_{max}}{2}$ (midpoint of the band)**
  - Carrier frequency depends only on the location of the band and not the bandwidth
- Bandwidth $B = f_{max} - f_{min}$
- **Choose bit rate as $B/2$ (half the bandwidth)**
  - Bit rate depends on the bandwidth and not the location of the band
  - $1/T_b = B/2 \implies T_b = 2/B$

Modulation

Input: Bit sequence $\{b_m\}$ for $m = 1, 2, \ldots$
Amplitude V
Bit duration T_b
Carrier Frequency f_c

1. Construct baseband signal $x(t)$ using on-off keying with amplitude V and bit duration T_b
2. Construct passband signal $s(t)$ using $s(t) = x(t) \times \sin(2\pi f_c t)$

Output: Waveform $s(t)$

Demodulation: Power Detection and Thresholding

1. Divide time into bit intervals of length T_b, i.e.,

$$[(m-1)T_b,\, mT_b], \qquad m = 1, 2, \ldots$$

2. For each bit interval, compute the average received power

$$p_m \;=\; \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} y^2(t)\,dt.$$

3. Compare the power to a threshold and decide the bit:

$$\hat{b}_m = \begin{cases} 1, & p_m > p_{\text{thresh}}, \\ 0, & p_m \le p_{\text{thresh}}. \end{cases}$$

Power Detection and Thresholding

- Suppose noiseless:
  - $y(t) = V \sin(2\pi f_c t)$ for $t \in [(m-1)T_b, mT_b]$ if $b_m = 1$
  - $y(t) = 0$ for $t \in [(m-1)T_b, mT_b]$ if $b_m = 0$
- What value to choose for p_{thresh}?
  - Assume T_b is divisible by $1/f_c$
    - The sinusoid completes an integer number of cycles over one bit duration
  - Given:

$$\int_0^{1/f_c} \sin^2(2\pi f_c t) = \frac{1}{2f_c}$$

    - Integral of square of sinusoid over one cycle of sinusoid is $1/(2f_c)$

Power Detection and Thresholding

Noiseless Case:

- When $b_m = 0$,

$$p_m = \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} y^2(t)\,dt = \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} 0\,dt = 0$$

- When $b_m = 1$,

$$\begin{aligned} p_m &= \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} y^2(t)\,dt = \frac{1}{T_b}\int_{(m-1)T_b}^{mT_b} V^2\sin^2(2\pi f_c t)\,dt \\ &= \frac{V^2}{T_b}\int_{(m-1)T_b}^{mT_b} \sin^2(2\pi f_c t)\,dt \end{aligned}$$

Integral Computation

$$\int_{(m-1)T_b}^{mT_b} \sin^2(2\pi f_c t)\, dt = (T_b f_c) \int_0^{1/f_c} \sin^2(2\pi f_c t)\, dt$$
$$= (T_b f_c) \cdot \frac{1}{2 f_c}$$
$$= \frac{T_b}{2}$$

- First equality: $\sin^2(2\pi f_c t)$ completes $T_b f_c$ cycles over the bit duration of $[(m-1)T_b, mT_b]$

Power Detection and Thresholding

Noiseless Case:

- When $b_m = 0$,

$$p_m = 0$$

- When $b_m = 1$,

$$\begin{aligned} p_m &= \frac{V^2}{T_b} \int_{(m-1)T_b}^{mT_b} \sin^2(2\pi f_c t)\, dt = \frac{V^2}{T_b} \cdot \frac{T_b}{2} \\ &= \frac{V^2}{2} \end{aligned}$$

Power Detection and Thresholding

Noiseless Case: We want to choose p_{thresh} such that:

- the received power ($\frac{V^2}{2}$) is larger than p_{thresh} when $b_m = 1$
  - will ensure $\hat{b}_m = 1$
- the received power (0) is smaller than p_{thresh} when $b_m = 0$
  - will ensure $\hat{b}_m = 0$

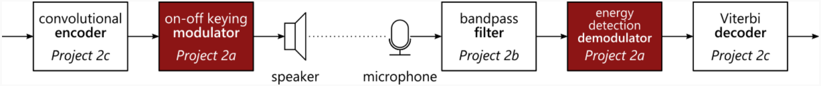
Answer: p_{thresh} can be chosen to be any value between 0 and $V^2/2$

- Will ensure error-free decoding

Project 2

Project 2

- Project 2 starts this week
- Communicating bits reliably over a noisy communication channel
  - Using audio signals
  - Noise: random noise, talking, clapping, etc. (simulated channels)

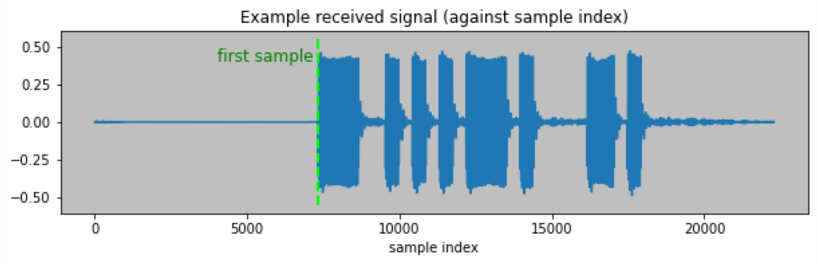


- Project 2a will be released tomorrow
  - Convert bits to audio signal (modulation)
  - Use power detection and thresholding (demodulation)

Audio Signals

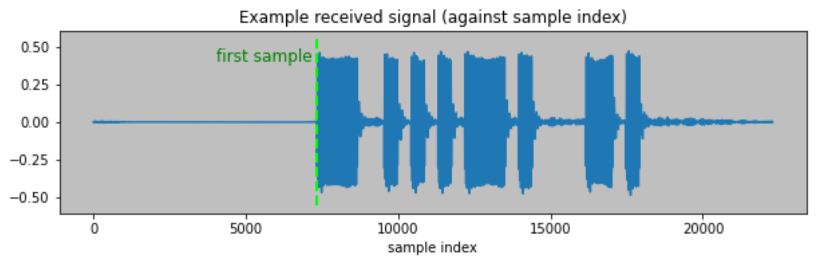
- We will generate signals with frequencies in our audible range
- Transmitted signal
- Received signal with random noise
- Received signal with talking
- Received signal with clapping

A Practical Challenge: Synchronization



- Received signal may start with unknown random delay
- Detect where *the actual message* starts in the received signal
- Without synchronization:
  - Bit boundaries are misaligned
  - Power detection is wrong and hence obtained bits are wrong

A Practical Challenge: Synchronization



- Simple scheme:
  - the first bit is always a 1
  - Check when the received signal crosses a threshold for the first time
  - Gives us the start of the message signal
- Better synchronization schemes - in lecture notes (optional)

Thank You!