Some formal analyses of determiners

Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2022

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This handout provides some model answers for the technical questions on Assignment 4. I hope also that it helps to further illuminate the determiner properties of intersectivity, conservativity, and monotonicity.

1 Non-intersectivity of *all but two*

Here is a possible (though not necessarily empirically correct) definition of the phrasal determiner *all but two* as in *all but two students passed*:

$$[\![\textit{all but two}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X - Y| = 2, \text{ else } \mathsf{F}\Big)\Big)$$

A determiner D is intersective iff $D(A)(B) = D(B)(A)$ for all A, B. This determiner is not intersective.

Consider $A = \{a, b, c\}$ and $B = \{a\}$. Then

$$[\![\textit{all but two}]\!](A)(B) = \mathsf{T} \text{ if } |A - B| = 2, \text{ else } \mathsf{F}$$

resolves to T, since $A - B = \{b, c\}$. However,

$$[\![\textit{all but two}]\!](B)(A) = \mathsf{T} \text{ if } |B - A| = 2, \text{ else } \mathsf{F}$$

resolves to F, since $B - A = \emptyset$, which has cardinality 0. For intersectivity, all it takes is one failure of entailment in one direction to establish that the determiner is not intersective, so our job is done.

2 Monotonicity of the first argument to *few*

Here is a possible (though not necessarily empirically correct) definition of the determiner $[\![\textit{few}]\!]$:

$$[\![\textit{few}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X \cap Y| < j, \text{ else } \mathsf{F}\Big)\Big)$$

where $j > 0$ is a pragmatic free variable (presumably set to a very small integer, though the size might depend on the nature of the first argument).

A determiner D is downward monotone on its first argument iff $D(A)(B)$ entails $D(X)(B)$ for all A, B, X where $X \subseteq A$. We can show that the first argument slot for $[\![\textit{few}]\!]$ is downward monotone.

Assume $[\![\textit{few}]\!](A)(B) = \mathsf{T}$ for arbitrary A and B, with j also set to some value. Then $|A \cap B| < j$ holds. Moving to a subset X of A can only make $|X \cap B| \leqslant |A \cap B|$, so truth is preserved no matter how j is set, and hence $[\![\textit{few}]\!](X)(B) = \mathsf{T}$.

3 Non-monotonicity of the first argument to *between 2 and 4*

Here's a proposed meaning for the phrasal determiner *between 2 and 4*:

$$[\![\textit{between 2 and 4}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } 2 \leqslant |X \cap Y| \leqslant 4, \text{ else } \mathsf{F}\big)\Big)$$

This determiner is nonmonotone on its first argument. Let $A = \{a, b, c, d, e\}$ and $B = \{b, c, d, e, f\}$. Then

$$[\![\textit{between 2 and 4}]\!](A)(B) = \mathsf{T} \text{ if } 2 \leqslant |A \cap B| \leqslant 4, \text{ else } \mathsf{F}$$

resolves to T because $A \cap B = \{b, c, d, e\}$, which has cardinality 4.

Now suppose we take $X = \{a, b, c, d, e, f\}$. This is a superset of A, but $X \cap B = \{b, c, d, e, f\}$, which has cardinality 5. This shows that the determiner is not upward monotone on the first argument.

Now suppose we set $X = \{b\}$. This is a subset of A, but $X \cap B = \{b\}$, which has cardinality 1. This shows that the determiner is not downward on the first argument.

Since $[\![\textit{between 2 and 4}]\!]$ is neither upward nor downward monotone on its first argument, we conclude that it is nonmonotone on its first argument.

4 Conservativity of *not every*

Here is a proposed meaning for the phrasal determiner *not every*;

$$[\![\textit{not every}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } X \nsubseteq Y, \text{ else } \mathsf{F}\big)\Big)$$

A determiner D is conservative iff $D(A)(B) = D(A)(A \cap B)$ for all A, B. This determiner is conservative.

To see this, first assume $[\![\textit{not every}]\!](A)(B) = \mathsf{T}$ for arbitrary sets A and B. Then we have that $A \nsubseteq B$. This means there is at least one x such that $x \in A$ but $x \notin B$. Any such x is also not in $A \cap B$ (because that would require $x \in B$), so $A \nsubseteq (A \cap B)$ holds, and thus $[\![\textit{not every}]\!](A)(A \cap B) = \mathsf{T}$.

For the other direction: assume $[\![\textit{not every}]\!](A)(A \cap B) = \mathsf{T}$. Then $A \nsubseteq (A \cap B)$ holds. This means there is at least one x such that $x \in A$ but $x \notin (A \cap B)$. Since we know $x \in A$, it must be that $x \notin B$, and thus we have $A \nsubseteq B$, which means $[\![\textit{not every}]\!](A)(B) = \mathsf{T}$.

5 A (non-existent) non-conservative determiner

Consider the hypothetical determiner $[\![\textit{somenon}]\!]$:

$$[\![\textit{somenon}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } ((U - X) \cap Y) \neq \emptyset, \text{ else } \mathsf{F}\big)\Big)$$

This hypothetical determiner is not conservative. To see this, we can just note that

$$[\![somenon]\!](A)(A \cap B) = \mathsf{T} \text{ if } ((U - A) \cap (A \cap B)) \neq \emptyset, \text{ else } \mathsf{F}$$

always resolve to F, since $(U - A) \cap A = \emptyset$ and this is preserved under intersection (of either side). Thus, any situation in which $[\![somenon]\!](A)(B)$ is true will work as a counterexample to conservativity. For example, suppose the universe $U = \{a, b\}$, $A = \{a\}$, and $B = \{b\}$. Then

$$[\![somenon]\!](A)(B) = \mathsf{T} \text{ if } \big(\{b\} \cap \{b\} \big) \neq \emptyset, \text{ else } \mathsf{F}$$

which resolves to T, but

$$[\![somenon]\!](A)(A \cap B) = \mathsf{T} \text{ if } \big(\{b\} \cap \{a\} \cap \{b\} \big) \neq \emptyset, \text{ else } \mathsf{F}$$

which resolves to F.