Introduction to the Rational Speech Acts model

Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2022

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1 Overview

This handout is meant to be a technical companion to our core RSA reading, Goodman & Frank 2016,[1] which provides relevant background, conceptual motivation, and model details. It's best to study that paper carefully before working with this handout. There's also a screencast available from the course website that provides a bit more conceptual background and walks through the calculations reviewed here.

2 Reference games

It's very useful to ground RSA calculations in specific reference games:

(1) A reference game is a structure $(R, M, [\![\cdot]\!], P, C)$, where

a. R is a set of states (worlds, referents, propositions, etc.).

b. M is a set of messages.

c. $[\![\cdot]\!] : M \mapsto R \mapsto \{0,1\}$ is a semantic interpretation function.

d. $P : R \mapsto [0,1]$ is a prior probability distribution over states.

e. $C : M \mapsto \mathbb{R}_{\leqslant 0}$ is a cost function on messages.

The intuitive idea here is that we have a speaker and a listener in a shared context/game. The speaker is privately assigned a *target* referent $r^* \in R$, and the speaker's goal is choose a message from M that will lead the listener to pick r^* as the target.

Implicitly, RSA assumes that the speaker and listener would both prefer for the listener to correctly identify r^*. This is a version of the cooperativity assumptions from Grice's work.

The guiding idea behind RSA is that these agents can do better at games like this by reasoning about each other rather than just about the truth conditions built into $[\![\cdot]\!]$. This connects very deeply with the definition of conversational implicature, which also centers around this back-and-forth reasoning.

[1] Goodman, Noah D. and Frank, Michael C. 2016. Pragmatic language interpretation as probabilistic inference. *Trends in Cognitive Sciences* 20(11): 818–829.

3 RSA and Grice

3.1 The RSA model

(2)

$$P_{\text{Lit}}(r \mid m) = \frac{[\![m]\!](r) \cdot P(r)}{\sum_{r' \in R} [\![m]\!](r') \cdot P(r')}$$

(3)

$$P_S(m \mid r) = \frac{\exp\left(\alpha \cdot (\log P_{\text{Lit}}(r \mid m) + C(m))\right)}{\sum_{m' \in M} \exp\left(\alpha \cdot (\log P_{\text{Lit}}(r \mid m') + C(m'))\right)}$$

(4)

$$P_L(r \mid m) = \frac{P_S(m \mid r) \cdot P(r)}{\sum_{r' \in R} P_S(m \mid r') \cdot P(r')}$$

3.2 Simplification where priors are flat, costs are 0, and $\alpha = 1$

(5)

$$P_{\text{Lit}}(r \mid m) = \frac{[\![m]\!](r)}{\sum_{r' \in R} [\![m]\!](r')}$$

(6)

$$P_S(m \mid r) = \frac{P_{\text{Lit}}(r \mid m)}{\sum_{m' \in M} P_{\text{Lit}}(r \mid m')}$$

(7)

$$P_L(r \mid m) = \frac{P_S(m \mid r)}{\sum_{r' \in R} P_S(m \mid r')}$$

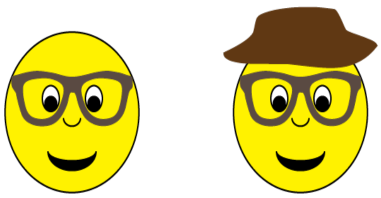
3.3 Connections to Grice

We don't *need* to reconstruct Grice's theory, but it's reassuring that we can make connections.

Grice	**RSA**
Quality	All agents assign 0 probability to false utterances.
Quantity	The speaker favors informative utterances.
Manner	The cost function C.
Relevance	Basic RSA doesn't engage this directly, though the referent prior helps.

The recursive nature of RSA aligns with the definition of conversational implicature (the speaker believes that the listener believes ...).

4 Simple scalar implicature



r_1 r_2

(a) Scenario

	r_1	r_2
'hat'	0	1
'glasses'	1	1

(b) $[\![\cdot]\!]$

r_1	0.5
r_2	0.5

(c) P

'hat'	0
'glasses'	0

(d) C

Figure 1: A communication game supporting a scalar implicature. For the calculations, $\alpha = 1$.

(8) a.

P_{Lit}	r_1	r_2
'hat'	0	1
'glasses'	0.5	0.5

$$P_{\text{Lit}}(r \mid m) = \frac{[\![m]\!](r)}{\sum_{r' \in R} [\![m]\!](r')}$$

b.

	'hat'	'glasses'
r_1	0	1
r_2	0.67	0.33

$$P_S(m \mid r) = \frac{P_{\text{Lit}}(r \mid m)}{\sum_{m' \in M} P_{\text{Lit}}(r \mid m')}$$

c.

	r_1	r_2
'hat'	0	1
'glasses'	0.75	0.25

$$P_L(r \mid m) = \frac{P_S(m \mid r)}{\sum_{r' \in R} P_S(m \mid r')}$$

Remarks This captures the scalar implicature pattern that a general term will tend to exclude any more specific salient terms. It's not clear whether this is an explanation based on quantity (informativity) or ambiguity avoidance (manner), but perhaps the distinction doesn't matter here!

Here are the calculations above in more detail:

(9) Start with the lexicon:

	r_1	r_2
'hat'	0	1
'glasses'	1	1

(10) Normalize the rows (divide each value by the sum of the values in its row):

P_{Lit}	r_1	r_2
'hat'	0	1
'glasses'	0.5	0.5

(11) Transpose the matrix so that the states are along the rows:

	'hat'	'glasses'
r_1	0	0.5
r_2	1	0.5

(12) Normalize the rows:

P_S	'hat'	'glasses'
r_1	0	1
r_2	0.67	0.33

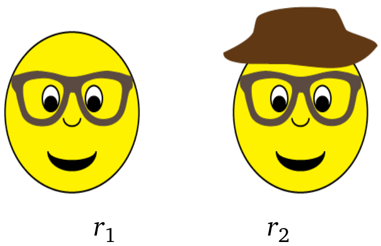
(13) Transpose:

	r_1	r_2
'hat'	0	0.67
'glasses'	1	0.33

(14) Normalize the rows:

P_L	r_1	r_2
'hat'	0	1
'glasses'	0.75	0.25

5 The role of message costs



(a) Scenario

	r_1	r_2
'hat'	0	1
'glasses'	1	1

(b) $[\![\cdot]\!]$

r_1	0.5
r_2	0.5

(c) P

'hat'	−6
'glasses'	0

(d) C

Figure 2: A communication game with very high costs on one message. For the calculations, $\alpha = 1$.

(15) a.

P_{Lit}	r_1	r_2
'hat'	0	1
'glasses'	0.5	0.5

$$P_{\text{Lit}}(r \mid m) = \frac{[\![m]\!](r) \cdot P(r)}{\sum_{r' \in R} [\![m]\!](r') \cdot P(r')}$$

b.

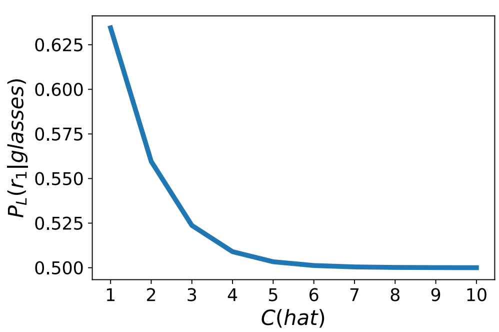
P_S	'hat'	'glasses'
r_1	0	1
r_2	0.0049	0.9951

$$P_S(m \mid r) = \frac{\exp(\alpha \cdot (\log P_{\text{Lit}}(r \mid m) + C(m)))}{\sum_{m' \in M} \exp(\alpha \cdot (\log P_{\text{Lit}}(r \mid m') + C(m')))}$$

c.

P_L	r_1	r_2
'hat'	0	1
'glasses'	0.5012	0.4988

$$P_L(r \mid m) = \frac{P_S(m \mid r) \cdot P(r)}{\sum_{r' \in R} P_S(m \mid r') \cdot P(r')}$$



Increasing the cost of a term (here 'hat') lets us model situations where that term is regarded as marked – perhaps too marked to be used. In such situations, that term doesn't compete with the terms it entails, so the implicature disappears. In the figure, we see that, by $C(\text{'hat'}) = -7$, P_L doesn't really treat 'glasses' as referring to r_1, because P_L expects 'glasses' to be the least marked way of referring to r_2 as well.

Figure 3: Exploring the space of cost functions.

Here are the calculations above in more detail:

(16) Start with the lexicon:

	r_1	r_2
'hat'	0	1
'glasses'	1	1

(17) Normalize the rows:

P_{Lit}	r_1	r_2
'hat'	0	1
'glasses'	0.5	0.5

(18) Transpose:

	'hat'	'glasses'
r_1	0	0.5
r_2	1	0.5

(19) Take the log of the values, subtract the costs, and exponentiate:

	'hat'	'glasses'
r_1	$\exp(\log(0) - 6)$	$\exp(\log(0.5) - 0)$
r_2	$\exp(\log(1) - 6)$	$\exp(\log(0.5) - 0)$

$\Rightarrow$

	'hat'	'glasses'
r_1	0	0.5
r_2	0.0025	0.5

(20) Normalize the rows:

P_S	'hat'	'glasses'
r_1	0	1
r_2	0.0049	0.9951

(21) Transpose:

	r_1	r_2
'hat'	0	0.0049
'glasses'	1	0.9951

(22) Normalize the rows:

P_L	r_1	r_2
'hat'	0	1
'glasses'	0.5012	0.4988

6 The role of the the alpha parameter

The alpha parameter can be seen as controlling how much pragmatics we see. Larger α results in stronger pragmatic inferences, and smaller α corresponds to weaker pragmatic inferences. Only the speaker agent uses α directly, but this means that the pragmatic listener is also affected by it.

Here's a version of the speaker with α but no cost terms, to simplify it a bit:

(23)

$$P_S(m \mid r) = \frac{\exp\left(\alpha \cdot (\log P_{\text{Lit}}(r \mid m))\right)}{\sum_{m' \in M} \exp\left(\alpha \cdot (\log P_{\text{Lit}}(r \mid m'))\right)}$$

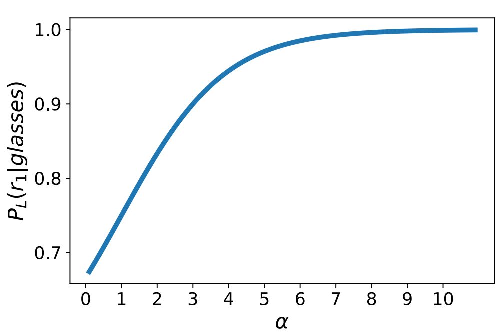
These two speaker matrices convey how large α amplifies the pragmatics:

(24) $\alpha = 1$

P_S	'hat'	'glasses'
r_1	0	1
r_2	0.67	0.33

(25) $\alpha = 4$

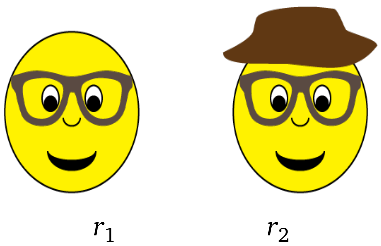
P_S	'hat'	'glasses'
r_1	0	1
r_2	0.94	0.06



As α gets bigger, the implicature gets stronger in the sense that P_L is more certain that 'glasses' must pick out r_1 even though it is also true of r_2. Where message costs are all the same, this reduces to the effect of multiplying α by the log of the P_{Lit} values, which affects the differences between values. For example, $1\log(0.75) - 1\log(0.25)$ is five times smaller than $5\log(0.75) - 5\log(0.25)$.

Figure 4: The effect of the rationality parameter α on the pragmatic listener.

7 The role of the referent prior



(a) Scenario

	r_1	r_2
'hat'	0	1
'glasses'	1	1

(b) $[\![\cdot]\!]$

r_1	0.3
r_2	0.7

(c) P

'hat'	0
'glasses'	0

(d) C

Figure 5: A communication game with skewed priors. For the calculations, $\alpha = 1$.

(26) a.

P_{Lit}	r_1	r_2
'hat'	0	1
'glasses'	0.3	0.7

$$P_{\text{Lit}}(r \mid m) = \frac{[\![m]\!](r) \cdot P(r)}{\sum_{r' \in R} [\![m]\!](r') \cdot P(r')}$$

b.

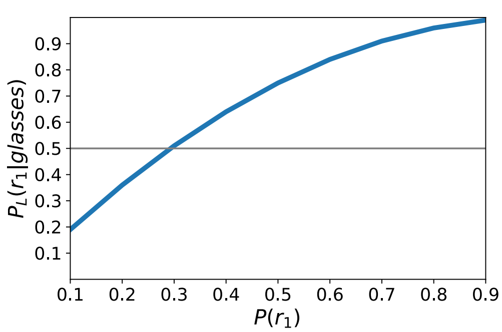
P_S	'hat'	'glasses'
r_1	0	1
r_2	0.59	0.41

$$P_S(m \mid r) = \frac{P_{\text{Lit}}(r \mid m)}{\sum_{m' \in M} P_{\text{Lit}}(r \mid m')}$$

c.

P_L	r_1	r_2
'hat'	0	1
'glasses'	0.51	0.49

$$P_L(r \mid m) = \frac{P_S(m \mid r) \cdot P(r)}{\sum_{r' \in R} P_S(m \mid r') \cdot P(r')}$$



Decreasing the prior on one referent models a situation in which that referent is unlikely in the view of the discourse participants. As this value gets smaller, this referent becomes less relevant, and expected implicatures can disappear. In the figure, we see this happen: if $P(r_1)$ is very low, hearing 'glasses' doesn't necessarily lead P_L to choose r_1 as a referent, because r_1 as a referent is so unlikely in general.

Figure 6: Exploring the space of referent priors.

Here are the calculations above in more detail:

(27) Start with the lexicon:

	r_1	r_2
'hat'	0	1
'glasses'	1	1

(28) Bring in the prior:

	r_1	r_2
'hat'	$0 \cdot 0.3$	$1 \cdot 0.7$
'glasses'	$1 \cdot 0.3$	$1 \cdot 0.7$

⇒

	r_1	r_2
'hat'	0	0.7
'glasses'	0.3	0.7

(29) Normalize the rows:

P_{Lit}	r_1	r_2
'hat'	0	1
'glasses'	0.3	0.7

(30) Transpose:

	'hat'	'glasses'
r_1	0	0.3
r_2	1	0.7

(31) Normalize the rows:

	'hat'	'glasses'
r_1	0	1
r_2	0.59	0.41

(32) Transpose:

P_S	r_1	r_2
'hat'	0	0.59
'glasses'	1	0.41

(33) Bring in the prior:

	r_1	r_2
'hat'	$0 \cdot 0.3$	$0.59 \cdot 0.7$
'glasses'	$1 \cdot 0.3$	$0.41 \cdot 0.7$

⇒

	r_1	r_2
'hat'	0	0.413
'glasses'	0.3	0.287

(34) Normalize the rows:

P_L	r_1	r_2
'hat'	0	1
'glasses'	0.51	0.49

[[•]]	r_1	r_2	r_3	C
'hat'	0	1	1	0
'glasses'	1	0	1	0
'mustache'	1	1	0	0
P	1/3	1/3	1/3	α = 1

P_{Lit}	r_1	r_2	r_3
'hat'	0	$\frac{1 * 1/3}{1 * 1/3 + 1 * 1/3}$	$\frac{1 * 1/3}{1 * 1/3 + 1 * 1/3}$
'glasses'	$\frac{1 * 1/3}{1 * 1/3 + 1* 1/3}$	0	$\frac{1 * 1/3}{1 * 1/3 + 1* 1/3}$
'mustache'	$\frac{1 * 1/3}{1 * 1/3 + 1* 1/3}$	$\frac{1 * 1/3}{1 * 1/3 + 1* 1/3}$	0

[[•]]	r_1	r_2	r_3	C
'hat'	0	1	1	0
'glasses'	1	0	1	0
'mustache'	1	1	0	0
P	0.10	0.30	0.60	$\alpha = 1$

P_{Lit}	r_1	r_2	r_3
'hat'	0	$\frac{1 * .3}{1 * .3 + 1 * .6}$	$\frac{1 * .6}{1 * .3 + 1 * .6}$
'glasses'	$\frac{1 * .1}{1 * .1 + 1 * .6}$	0	$\frac{1 * .6}{1 * .1 + 1 * .6}$
'mustache'	$\frac{1 * .1}{1 * .1 + 1 * .3}$	$\frac{1 * .3}{1 * .1 + 1 * .3}$	0

P_{Lit}	r_1	r_2	r_3	C
'hat'	0	0.44	0.56	–3
'glasses'	0.17	0	0.83	–2
'mustache'	0.20	0.80	0	–1
P	0.10	0.40	0.50	$\alpha = 1$

P_S	'hat'	'glasses'	'mustache'
r_1	0	$\frac{\exp(\log(.17) - 2)}{\exp(\log(.17) - 2) + \exp(\log(.2) - 1)}$	$\frac{\exp(\log(.2) - 1)}{\exp(\log(.17) - 2) + \exp(\log(.2) - 1)}$
r_2	$\frac{\exp(\log(.44) - 3)}{\exp(\log(.44) - 3) + \exp(\log(.8) - 1)}$	0	$\frac{\exp(\log(.8) - 1)}{\exp(\log(.44) - 3) + \exp(\log(.8) - 1)}$
r_3	$\frac{\exp(\log(.56) - 3)}{\exp(\log(.56) - 3) + \exp(\log(.83) - 2)}$	$\frac{\exp(\log(.83) - 2)}{\exp(\log(.56) - 3) + \exp(\log(.83) - 2)}$	0

P_S	'hat'	'glasses'	'mustache'	P
r_1	0	0.17	0.83	0.1
r_2	0.06	0	0.94	0.3
r_3	0.22	0.78	0	0.6
C	–3	–2	–1	$\alpha = 1$

P_L	r_1	r_2	r_3
'hat'	0	$\frac{.06 * .3}{.06 * .3 + .22 * .6}$	$\frac{.22 * .6}{.17 * .3 + .22 * .6}$
'glasses'	$\frac{.17 * .1}{.17 * .1 + .78 * .6}$	0	$\frac{.78 * .6}{.17 * .1 + .78 * .6}$
'mustache'	$\frac{.83 * .1}{.83 * .1 + .94 * .3}$	$\frac{.94 * .3}{.83 * .1 + .94 * .3}$	0