Set theory basics: answer key

1 Exercises

(1) The following sets are represented in the predicate notation. Convert them to the enumeration notation.

a. {3v: v is an even prime number}

Answer: {6}

Explanation: 2 is the only even prime number. 3(2) is 6. Therefore, the set in the enumeration notation is: {6}

b. {z: z is an integer and $z > 0$ and $z < 10$ and z^2 is a prime number}

Answer: $\varnothing$

Explanation: Is there an integer between 0 and 10 such that its square is a prime number? There isn't. Therefore, the set in the enumeration notation is: $\varnothing$

NB: 1 is not considered a prime number

c. {z: z is an integer and $z > 0$ and $z < 5$ and Barack Obama is a former US president}

Answer: {1, 2, 3, 4}

Explanation: This is pretty straightforward. That Barack Obama is a former US president doesn't say anything about what should belong to the set.

(2) The following sets are represented in the enumeration notation. Convert them to the predicate notation.

Note that there are multiple ways of defining these sets via the predicate notation. We have provided one correct definition for each of these sets.

a. $\{1, 2, 3, 4\}$

Answer: $\{x : x$ is an integer and $0 < x < 5\}$

b. $\{2, 3, 5, 7\}$

Answer: $\{x$: x is a prime number and $x < 8\}$

c. {Barack Obama, Donald Trump, Joe Biden}

Answer: $\{x$: x has been a US president at some point between 2008 and 2024$\}$

(3) Calculate the cardinality of each of the following sets:

a. $\{1, 2, 3, 4\}$

Answer: 4

b. $\{1, 2, \{3, 4\}, 0\}$

Answer: 4

c. $\varnothing$

Answer: 0

d. $\{\varnothing\}$

Answer: 1

e. $\{\{1, 2\}\}$

Answer: 1

f. $\{\{1, 2, \{3, 4\}, 5\}, \{6, 7\}, 8\}$

Answer: 3

g. {3v: v is an even prime number}

Answer: 1

(4) Let A be an arbitrary set. Are the following always true?

a. $\varnothing \subset A$

Answer: This is false. A can be the empty set itself.

b. $\varnothing \subseteq A$

Answer: This is true. $\varnothing$ is a subset of every set. Note that while it is true that $\varnothing$ is a subset of itself, it is false that $\varnothing$ is a proper subset of itself.

c. $\varnothing \in A$

Answer: This is false. A can be the set $\{1, 2, 3\}$. Then, A wouldn't contain $\varnothing$.

d. $A = \{y : y \in A\}$

Answer: This is true. A does contain all y such that y is in A.

e. $A = \{y : y \subseteq A\}$

Answer: This is false. A can be the set $\{1\}$. 1 is not a set. Therefore, it is not the case that $1 \subseteq A$; although 1 is in A.

(5) Write out the power set of each of the following sets:

a. $\{1, 2\}$

Answer: $\{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$

b. $\{1, 2, \{3, 4\}, 0\}$

Answer: $\{\varnothing$,{1},{2},{{3,4}},{0},
{1,2},{1,0},{1,{3,4}},
{2,0},{2,{3,4}},
{0,{3,4}},
{1,2,{3,4}},{1,0{3,4}},{1,2,0}
{0,2,{3,4}},
{1,2,{3,4},0}$\}$

c. $\varnothing$

Answer: $\{\varnothing\}$

d. $\{\varnothing\}$

Answer: $\{\varnothing,\{\varnothing\}\}$

(6) Calculate the following, writing out the answers in enumeration notation:

a. $\{1, 0, 3\} \cup \{1, 2, 3\}$

Answer: $\{0,1,2,3\}$

b. $\{1, 0, 3\} \cap \{2, 4\}$

Answer: $\varnothing$

c. $\{0, 1, 5\} - \{0, 1\}$

Answer: $\{5\}$

d. $\{0, 1, 5\} - \{2, 5, 8\}$

Answer: $\{0,1\}$

e. $\{y$: y is not $3\}^{\complement}$

Answer: $\{3\}$

(7) Suppose $A - B = \varnothing$. Are the following statements true?

a. $A \cup B = B$

Answer: This is true. If $A - B$ is empty, then there is no element in A such that it is not in B. Therefore, the union of B with A is not greater than B and contains all members of B.

b. $A \cap B = A$

Answer: This is true. If $A - B$ is empty, then there is no element in A such that it is not in B. Therefore, the intersection of A with B is not greater than A and contains all members of A, as they are also contained in B.