1. Classify the following in terms of degree of nonlinearity:
   (a) \( u_t^2 + x^2u_{xt} = \sin(u) \)
   (b) \( u_x + [u^3]_y = x^2 + y^2 \)
   (c) \([e^u]_x + u^2u_y = 0\)
   (d) \([x^3u]_x + y^3u = \sin(x^2 + y^2)\)
   (e) \([u^3]_x + e^{ux} = 0\)

2. Solve
\[
\begin{cases}
  u_t + u_x^2 = t \\
  u(x,0) = x.
\end{cases}
\]

3. Solve
\[
\begin{cases}
  u_{tt} + 3u_{xt} - 10u_{xx} = 0 \\
  u(x,0) = \phi(x) \\
  u_t(x,0) = \psi(x)
\end{cases}
\]
by reducing the hyperbolic equation to two first-order transport equations. That is, reduce to the system
\[
\begin{aligned}
  (\partial_t + 5\partial_x)v &= 0 \\
  (\partial_t - 2\partial_x)u &= v
\end{aligned}
\]
with appropriate initial conditions. Then solve these first-order equations using the method of characteristics.

4. Find the unique, weak solution of the following which satisfies the entropy condition,
\[
\begin{cases}
  u_t - (\sin(u))_x = 0 \quad t \geq 0 \\
  u(x,0) = \phi(x)
\end{cases}
\]
in each of the two cases below:
(a) \( \phi(x) = \begin{cases} 
0 & x < 0 \\
\pi & x > 0.
\end{cases} \)
(b) \( \phi(x) = \begin{cases} 
\pi & x < 0 \\
0 & x > 0.
\end{cases} \)
5. We say \( u \) is a weak solution of

\[
\begin{aligned}
\{ & [g(u)]_t + [f(u)]_x = 0 \\
& u(x, 0) = \phi(x)
\end{aligned}
\]

if \( u \) satisfies

\[
\int_0^\infty \int_{-\infty}^\infty g(u)v_t + f(u)v_x \, dx \, dt + \int_{-\infty}^\infty \phi(x)v(x, 0) \, dx = 0
\]

for all \( v \in C^\infty(\mathbb{R} \times [0, \infty)) \) with compact support. Suppose \( u \) is a weak solution of (*) such that \( u \) has a jump discontinuity across the curve \( x = \xi(t) \), but \( u \) is smooth on either side of the curve \( x = \xi(t) \). Let \( u^-(x, t) \) be the value of \( u \) to the left of the curve and \( u^+(x, t) \) be the value of \( u \) to the right of the curve. Prove that \( u \) must satisfy the condition

\[
\frac{[f(u)]}{[g(u)]} = \xi'(t)
\]

across the curve of discontinuity, where

\[
\begin{aligned}
[f(u)] &= f(u^-) - f(u^+) \\
[g(u)] &= g(u^-) - g(u^+).
\end{aligned}
\]