

Math 220A - Fall 2002
Homework 1
Due Friday, Oct. 4, 2002

1. Classify the following equations in terms of degree of nonlinearity: linear, semilinear, quasilinear, fully nonlinear.

(a) $u_t + u_x + \sin(u) = 0$

(b) $u_t + u_x + \sin(x^2) = 0$

(c) $u_t + u_x + \sin(u_x) = 0$

(d) $u_t + e^u = x^2 u^2$

(e) $u_t + e^u u_x = \sin(x^2)$

2. Solve

$$\begin{cases} u_t + xu_x = t^3 \\ u(x, 0) = \phi(x). \end{cases}$$

3. Solve

$$\begin{cases} u_t + xu_x = u^3 \\ u(x, 0) = \sin(x). \end{cases}$$

At some time $T > 0$, the solution u blow up. That is, there exist points x_0 such that $|u(x_0, T)| \rightarrow +\infty$. Find the smallest time T , and the points x_0 such that $|u(x_0, t)| \rightarrow +\infty$ as $t \rightarrow T^-$.

4. (a) Show there are no solutions to

$$\begin{cases} xu_t + u_x = 0 \\ u(x, 0) = \sin(x). \end{cases}$$

- (b) Explain why there are an infinite number of solutions of

$$\begin{cases} xu_t + u_x = 0 \\ u(x, 0) = \cos(x) \end{cases}$$

5. Solve

$$\begin{cases} u_t + uu_x = 0 \\ u(x, 0) = \sin(x). \end{cases}$$

Find the time $T > 0$ such that $u(x, t)$ is smooth for $0 \leq t < T$ and $u_x(x, t)$ becomes infinite at time T for some $x = x_0$. Find x_0 .