1. Classify the following equations in terms of degree of nonlinearity: linear, semilinear, quasilinear, fully nonlinear.
   
   (a) \( u_t + u_x + \sin(u) = 0 \)
   
   (b) \( u_t + u_x + \sin(x^2) = 0 \)
   
   (c) \( u_t + u_x + \sin(u_x) = 0 \)
   
   (d) \( u_t + e^u = x^2u^2 \)
   
   (e) \( u_t + e^uu_x = \sin(x^2) \)

2. Solve
   
   \[
   \begin{cases}
   u_t + xu_x = t^3 \\
   u(x, 0) = \phi(x).
   \end{cases}
   \]

3. Solve
   
   \[
   \begin{cases}
   u_t + xu_x = u^3 \\
   u(x, 0) = \sin(x).
   \end{cases}
   \]

   At some time \( T > 0 \), the solution \( u \) blow up. That is, there exist points \( x_0 \) such that \( |u(x_0, T)| \to +\infty \). Find the smallest time \( T \), and the points \( x_0 \) such that \( |u(x_0, t)| \to +\infty \) as \( t \to T^- \).

4. (a) Show there are no solutions to
   
   \[
   \begin{cases}
   xu_t + u_x = 0 \\
   u(x, 0) = \sin(x).
   \end{cases}
   \]

   (b) Explain why there are an infinite number of solutions of

   \[
   \begin{cases}
   xu_t + u_x = 0 \\
   u(x, 0) = \cos(x)
   \end{cases}
   \]

5. Solve
   
   \[
   \begin{cases}
   u_t + uu_x = 0 \\
   u(x, 0) = \sin(x).
   \end{cases}
   \]

   Find the time \( T > 0 \) such that \( u(x, t) \) is smooth for \( 0 \leq t < T \) and \( u_x(x, t) \) becomes infinite at time \( T \) for some \( x = x_0 \). Find \( x_0 \).