Math 220A - Fall 2002
Homework 7
Due Friday, November 22, 2002

1. Suppose that the series
$$\sum_{n=1}^{\infty} f_n(x)$$
converges uniformly on $[a, b]$ to $f(x)$. Show that
$$\sum_{n=1}^{\infty} \int_a^b f_n(x)\, dx = \int_a^b f(x)\, dx$$
provided f_n and f are integrable on $[a, b]$.

2. Suppose that f is in $C^2([-L, L])$ and satisfies $f(L) = f(-L)$.
   (a) Show that, for some positive constant C, the Fourier coefficients satisfy
   $$|A_n| \leq \frac{C}{n^2} \qquad \text{and} \qquad |B_n| \leq \frac{C}{n^2}$$
   for all integers $n \geq 1$.
   (b) Show that the Fourier series for f converges absolutely at each point x in $[-L, L]$. Note: You do not need to prove convergence to $f(x)$.

3. (a) Calculate the Fourier sine series for $f(x) = \cos(x)$ on the interval $[0, \pi]$.
   (b) Justify that the Fourier sine series obtained in part (a) converges to $\cos(x)$ pointwise on $(0, \pi)$ but not uniformly on $[0, \pi]$.
   (c) Show that term-by-term differentiation fails.

4. Solve
$$\begin{cases} u_{tt} - c^2 u_{xx} = e^t \sin(5x) & 0 < x < \pi \\ u(x, 0) = 0 \\ u_t(x, 0) = \sin(3x) \\ u(0, t) = 0 = u(\pi, t) \end{cases}$$

5. Solve
$$\begin{cases} u_{tt} - u_{xx} = 0 & 0 < x < \pi \\ u(x, 0) = 0 \\ u_t(x, 0) = 0 \\ u(0, t) = g(t) \\ u_x(\pi, t) + u(\pi, t) = h(t) \end{cases}$$