Math 220B **Midterm Exam** **July 25, 2002**

Julie Levandosky

Name: ______________________________

Please sign below in acknowledgment and acceptance of the Honor Code.

Signature: ______________________________

This exam is closed notes, closed book. The exam is worth a total of 60 points. The point value of each problem is indicated. Please show all work and clearly mark your answer.

Number	Points
1.	
2.	
3.	
4.	
5.	
6.	
Total	

1. (8 points) Let $\phi \in L^2(\mathbb{R})$. Use the Fourier transform to solve

$$\begin{cases} u_t - (\sin t)u_x = 0 & -\infty < x < \infty,\ t > 0 \\ u(x,0) = \phi(x). \end{cases}$$

2. (12 points) Let Ω be the upper half of the unit disk in $\mathbb{R}^2$. That is, let

$$\Omega \equiv \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1, y > 0\}.$$

Solve

$$\begin{cases} u_{xx} + u_{yy} = 0 & (x,y) \in \Omega \\ u(r,0) = 0 = u(r,\pi) \\ u(1,\theta) = \theta(\theta - \pi). \end{cases}$$

3. (6 points) Let Ω be an open, bounded subset of $\mathbb{R}^n$. Let $\alpha > 0$. Prove uniqueness of solutions to the following problem,

$$\begin{cases} \Delta u - \alpha u = f & x \in \Omega \\ \frac{\partial u}{\partial \nu} = g & x \in \partial\Omega \end{cases}$$

4. (12 points) Solve the following initial/boundary value problem,

$$\begin{cases} u_t - u_{xx} = 0 & 0 < x < \pi,\ t > 0 \\ u(x,0) = \phi(x) & 0 < x < \pi \\ u_x(0,t) = g(t) & t > 0 \\ u(\pi,t) = h(t) & t > 0. \end{cases}$$

5. (10 points) Let Ω be an open, bounded subset of $\mathbb{R}^2$. Let $f : \mathbb{R}^2 \to \mathbb{R}$. Assume $f(x, y) \geq 0$ for all $(x, y) \in \Omega$. Suppose u is a solution of Poisson's equation

$$\begin{cases} u_{xx} + u_{yy} = f(x, y) & (x, y) \in \Omega \subset \mathbb{R}^2 \\ u(x, y) = g(x, y) & (x, y) \in \partial\Omega. \end{cases}$$

Show that

$$\max_{\overline{\Omega}} u(x, y) = \max_{\partial\Omega} g(x, y).$$

6. (12 points) Determine whether the following statements are true or false. **Provide a reason for your answer.**

(a) Let Ω be an open, bounded subset of $\mathbb{R}^n$. Let $\Omega^c \equiv \mathbb{R}^n \backslash \Omega$. There exists at most one solution of

$$\begin{cases} \Delta u = 0 & x \in \Omega^c \\ u = g & x \in \partial\Omega \end{cases}$$

(b) Let Ω be an open, bounded subset of $\mathbb{R}^n$. Suppose u is a solution of

$$\begin{cases} u_t - \Delta u = 0 & x \in \Omega,\ t > 0 \\ u(x,0) = \phi(x) & \\ u(x,t) = 0 & x \in \partial\Omega. \end{cases}$$

Then

$$\int_\Omega u^2(x,t)\,dx = \int_\Omega \phi^2(x)\,dx.$$

(c) Let $\phi : \mathbb{R}^n \to \mathbb{R}$ be a bounded function. There exits at most one bounded solution u of

$$\begin{cases} u_t - \Delta u = 0 & x \in \mathbb{R}^n,\ t > 0 \\ u(x,0) = \phi(x). \end{cases}$$

(d) If u is a harmonic function on the rectangle

$$\Omega \equiv \{(x,y) \in \mathbb{R}^2 : 0 < x < a, 0 < y < b\},$$

then

$$\int_0^a u_y(x,0)\,dx + \int_0^b u_x(a,y)\,dy + \int_0^a u_y(x,b)\,dx + \int_0^b u_x(0,y)\,dy = 0.$$

Scratch Paper