Math 220B **Midterm Exam** **February 19, 2002**

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Name: ______________________________

Please sign below in acknowledgment and acceptance of the Honor Code.

Signature: ______________________________

This exam is closed notes, closed book. The exam is worth a total of 50 points. Each problem is worth 10 points. Please show all work and clearly mark your answer.

Number	Points
1.	
2.	
3.	
4.	
5.	
Total	

1. Use the Fourier transform to solve

$$\begin{cases} u_t = ku_{xx} - u & x \in \mathbb{R}, t > 0 \\ u(x,0) = \phi(x). \end{cases}$$

Express your answer in the form

$$u(x,t) = \int_{-\infty}^{\infty} K(x,y,t)\phi(y)\,dy$$

for some kernel $K(x,y,t)$. You may use the fact that for $x \in \mathbb{R}$,

$$f(x) = e^{-ax^2} \implies \widehat{f}(\xi) = \frac{1}{\sqrt{2a}} e^{-\xi^2/4a}.$$

2. Show that $u(x) = e^{-|x|}$ solves

$$-u_{xx} + u = 2\delta_0$$

in the sense of distributions.

3. (a) Let $\Omega = \{(x,y) \in \mathbb{R}^2 : 0 < x < \pi, 0 < y < \pi\}$. Solve the following boundary-value problem,

$$\begin{cases} \Delta u = 0 & (x,y) \in \Omega \\ u(0,y) = 0 = u(\pi,y) & 0 < y < \pi \\ u(x,0) = 0 & 0 < x < \pi \\ u(x,\pi) = 2\sin(x) - \sin(2x) & 0 < x < \pi. \end{cases}$$

(b) Find $\max_{\overline{\Omega}} u(x,y)$.

4. (a) Solve

$$\begin{cases} u_t - u_{xx} + u = f(x) & 0 < x < l \\ u(x,0) = g(x) & 0 < x < l \\ u_x(0,t) = 0 = u_x(l,t). \end{cases}$$

(b) Prove uniqueness of this solution.

5. (a) Answer true or false to the following statements. No explanation is necessary. (1 point each)

   i. Let Ω be an open, bounded subset of $\mathbb{R}^n$. Assume g satisfies the compatibility condition
   $$\int_{\partial\Omega} g(y)\, dS(y) = 0.$$
   There exists at most one smooth solution of
   $$\begin{cases} \Delta u = 0 & x \in \Omega \\ \frac{\partial u}{\partial \nu} = g & x \in \partial\Omega. \end{cases}$$

   ii. If u is a bounded, harmonic function on $\mathbb{R}^n$, then u must be constant.

   iii. There exists at most one smooth, bounded solution of
   $$\begin{cases} u_t - k\Delta u = f & x \in \mathbb{R}^n, t > 0 \\ u(x,0) = \phi(x) \end{cases}$$

   iv. If u is a smooth, harmonic function in Ω and $u = g \geq 0$ on $\partial\Omega$, then $u > 0$ in Ω.

(b) Answer the following short answer questions. (3 points each)

i. What is the derivative of the delta function?

ii. State the strong maximum principle for the heat equation.