Math 41: Calculus
Second Exam — November 13, 2007

Name: ________________________________

Section Leader: David Xiannan Jason Tracy Ziyu
(Circle one) Fernandez-Duque Li Lo Nance Zhang

Section Time: 11:00 1:15
(Circle one)

• This is a closed-book, closed-notes exam. No calculators or other electronic devices will be permitted. You have 2 hours.

• In order to receive full credit, please show all of your work and justify your answer. You do not need to simplify your answers unless specifically instructed to do so.

• If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

• It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until Tuesday, December 4, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

• Please sign the following:

   “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

   Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

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1. (15 points) Differentiate, using the method of your choice.

(a) \( f(x) = e^{2x-7} + \sqrt{1 - \sqrt{1 - x^4}} \)

(b) \( g(x) = \left( \frac{1}{3} \right)^x + \ln(x - \arcsin x) \)

(c) \( h(x) = (1 + x^2)^{\arctan x} \)
2. (10 points) The equation \( x^2 - xy + y^2 = 1 \) describes an ellipse.

(a) Find an expression for \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).

(b) Find the equation of the tangent line at the point (1, 1).

(c) Find the coordinates \((x, y)\) of all points on the curve where the tangent to the curve is horizontal.
3. (9 points)

(a) Find the linearization of the function $f(x) = x^{2/3}$ at the point $a = 8$; that is, find the linear function $L(x)$ that best approximates $f(x)$ for values of $x$ near 8.

(b) Use the linearization to estimate $(8.03)^{2/3}$. Is your approximation an overestimate or underestimate of the actual value? Explain fully.
4. (10 points) Dennis, a 5-foot-tall man, notices a small spaceship on the ground, located 40 feet from where he stands in a flat field. The spaceship suddenly begins a rapid vertical ascent, at a rate of 10 feet per second. Throughout the ascent, a bright light on the ship illuminates the entire field below, casting a shadow of Dennis onto the ground. What is the rate of change of the length of Dennis’s shadow exactly three seconds after the spaceship has taken off? (Hint: at any moment, the head of Dennis’s shadow is always located on the ground, and on the line determined by the ship’s light and Dennis’s head.)
5. (10 points) Consider the function \( f(t) = \frac{t}{3} + \sqrt{5 - t} \).

(a) Find all critical numbers of \( f \). Show all reasoning.

(b) Find the absolute maximum and minimum values of \( f \) on the interval \([0, 13]\).
6. (17 points) Consider the function \( h(x) = x^2 \ln x \).

(a) State the domain of \( h \).

(b) Determine if \( h \) has any asymptotes; your reasoning must utilize limits to determine the behavior of \( h \) at each end of the domain you found in (a).

(c) On what interval(s) is \( h \) increasing? decreasing? Explain completely.
(d) On what interval(s) is $h$ concave up? concave down? Explain completely.

(e) Using the information you’ve found, sketch the graph $y = h(x)$. Label and provide the $(x, y)$ coordinates of all local extrema and inflection points.
7. (10 points) Compute the following limits, showing all reasoning.

(a) \( \lim_{x \to \pi/2} \frac{(2x - \pi)^2}{\cos 2x + 1} \)

(b) \( \lim_{x \to 0^+} (1 + 3x)^{1/x} \)
8. (10 points) A landscape architect plans to enclose a 4000-square-foot rectangular region in a botanical garden. She will use shrubs costing $20 per foot along three sides and fencing costing $5 per foot along the fourth side. Find and justify the minimum total cost.
9. (9 points) The picture below depicts the graph of the function \( f(x) = -x^3 - 2x^2 + x + 3 \).

\[
\begin{align*}
\text{(a) } & \text{Suppose that Newton's method is used to approximate the value of the positive zero-crossing (root) of } f \text{ with initial approximation } x_1 = a. \text{ Use the expression above for } f \text{ to find a formula that gives the value of the subsequent estimate } x_2. \text{ Your answer should be given as a rational expression involving } a \text{ alone.} \\
\text{(b) } & \text{Suppose the initial guess is } -1; \text{ on the graph above, draw the tangent line that you would use to find the subsequent estimate } x_2 \text{ for the root. Then use your formula to find the value of } x_2; \text{ simplify your answer as much as you can. What do your results suggest about the usefulness of Newton's method for this initial guess?} \\
\text{(c) } & \text{Repeat all aspects of part (b) if the initial guess is } 1. \text{ (Use the same diagram above, and give your calculations and conclusions below.)}
\end{align*}
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