Math 41: Calculus
First Exam — October 14, 2008

Name: ____________________________________________________

Section Leader: ___________________________________________
(Circle one) Bob Hough Joe Rabinoff David Sher Nathan Stiennon Ian Weiner

Section Time: __________________________
(Circle one) 11:00 1:15

• Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.

• You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.

• You have 2 hours. This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.

• Please check that your copy of this exam contains 13 pages and is correctly stapled.

• If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

• It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until Tuesday, October 28, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

• Please sign the following:

  “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

  Signature: ____________________________________________

The following boxes are strictly for grading purposes. Please do not mark.

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1. (18 points) Find each of the following limits, with justification. If there is an infinite limit, then explain whether it is $\infty$ or $-\infty$.

(a) \( \lim_{x \to 5} \frac{1 - 5/x}{x^2 - 25} \)

(b) \( \lim_{x \to \infty} \sqrt{x^2 + 8x - x} \)
(c) \[ \lim_{t \to -3^+} \frac{t - 3}{t^2 + t - 6} \]

(d) \[ \lim_{x \to 0} x^4 e^{|\cos(1/x)|} \]
2. (5 points) Show that \( \lim_{x \to -3} (10 + 2x) = 4 \) by finding a \( \delta > 0 \) such that

\[
| (10 + 2x) - 4 | < \epsilon \quad \text{whenever} \quad 0 < |x - (-3)| < \delta.
\]
3. (14 points) Let $f(x) = \frac{1}{1 + e^{1/x}}$.

(a) Find the domain of $f$.

(b) Find the equations of all vertical asymptotes of $f$, or explain why none exist. As justification for each asymptote $x = a$, calculate both the one-sided limits $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$, showing your reasoning.
(c) Find the equations of all horizontal asymptotes of $f$, or explain why none exist. Justify using limit computations.

(d) It is a fact that $f$ is a one-to-one function. Find an expression for the function $f^{-1}(x)$, the inverse of $f$.

(e) Find the range of $f$. 
4. (8 points) Let \( f(x) = \frac{x}{4 - x^2} \). Find a formula for \( f'(x) \) using the limit definition of the derivative. Show the steps of your computation.
5. (6 points) Which of the functions below could be the derivative of which of the others? List as many correct answers as possible. No justification is necessary.
6. (9 points)

(a) Complete the following definition in precise terms: a function \( f(x) \) is said to be \textit{continuous} at the point \( x = a \) if

(b) Suppose \( g(x) = \begin{cases} x^2 - 1 & \text{if } x \neq 1, -1 \\ \frac{|x| - 1}{2} & \text{otherwise.} \end{cases} \)

Find all points where \( g \) is continuous, and all points where \( g \) is \textit{not} continuous, and give complete reasoning.
7. (16 points) Find the derivative, using any method you like. You do not need to simplify your answers.

(a) \( f(x) = \frac{2x^2}{3} + \frac{2}{3x^2} - \frac{2}{3} \)

(b) \( h(t) = \frac{t^3 - \cos t}{t + 1} \)

(c) \( Q(x) = \frac{(17 + \sqrt{x^2})(x + x^{-2})}{x^2} \)

(d) \( g(t) = \sqrt{t} e^t \sin t \)
8. (6 points) Using the graph below of $y = f(x)$, list the following quantities in increasing order (from smallest to largest). No justification is necessary.

$$f'(2) \quad f(0) \quad f'(-0.9) \quad \text{The number 1} \quad f''(0) \quad f''(1)$$
9. (7 points) A struggling airline company considers charging a fee for each checked bag brought on board by customers, and it asks industry consultants for market research on how such a fee will affect ticket sales. Let \( T(x) \) be the number of tickets per flight that the company can sell, if it sets the fee to \( x \) dollars per bag.

\[
\begin{array}{|c|c|c|c|c|}
\hline
x & 15 & 20 & 25 & 30 & 35 \\
\hline
T(x) & 132 & 128 & 122 & 115 & 107 \\
\hline
\end{array}
\]

(a) Estimate the value of \( T'(25) \). What are its units?

(b) What is the practical meaning of the quantity \( T'(25) \)? Give a brief but specific one- or two-sentence explanation that is understandable to someone who is not familiar with calculus.
10. (11 points) The following is a graph of $g'$, the derivative of some differentiable function $g$.

Use the above graph of $g'$ to give brief answers to the following questions about the function $g$. You 
do not need to provide justification for your answers. (But be careful: the above is a graph of the 
derivative of $g$, not of $g$ itself!)

(a) On what intervals is $g$ increasing? decreasing?

(b) On what intervals is $g$ concave up? concave down?

(c) Now suppose that $g(0) = 0$. Sketch a plausible graph of $g$ on the set of axes below. Label the 
vertical scale you use, and clearly label the locations of horizontal tangents ("HT") and points 
of inflection ("IP").