• Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.

• You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.

• You have 2 hours. This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.

• Please check that your copy of this exam contains 12 pages and is correctly stapled.

• If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

• It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until Tuesday, December 2, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

• Please sign the following:

  “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

  Signature: __________________________________________

The following boxes are strictly for grading purposes. Please do not mark.

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1. (15 points) Differentiate, using the method of your choice.

(a) \( f(x) = 2^{x^2+x} - \sin^2(2x + 3) \)

(b) \( g(t) = \sqrt{\ln t} + \arctan \sqrt{t} \)

(c) \( h(x) = \frac{|x| \cos x}{(x^2 + 3)^5} \)
2. (10 points) The equation \( x^{2/3} + y^{2/3} = 10 \) describes a curve in the \( xy \)-plane.

(a) Find an expression for \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).

(b) Find the equation of the line tangent to the curve at the point \((27, 1)\).

(c) Find the coordinates \((x, y)\) of all points on the curve where the tangent line is parallel to the line \( y = -x \).
3. (7 points) A cube is measured (via a water-displacement process) to have a volume of $1000 \text{ cm}^3$, with a possible error of $1 \text{ cm}^3$. Suppose we use this volume measurement to calculate the side-length of the cube. Use linear approximations (or differentials) to estimate the maximum error in the calculated side length.
4. (10 points) During an air show, a stunt jet is on a heading to fly directly over a stationary observer. The jet is traveling at 0.2 km/sec and maintaining a constant altitude of 3 km above the observer’s head. Meanwhile, the observer is tilting a camera further and further upward in order to keep it pointed at the moving jet. How fast is the angle that the camera makes with the ground changing exactly 10 seconds before the jet is directly overhead?
5. (8 points) Find the absolute minimum and maximum values of the function

\[ h(x) = 3x^4 + 8x^3 - 18x^2 + 7 \]

on the interval \([-1, 2]\). Show all reasoning.
6. (17 points) Consider the function \( g(x) = x(\ln x)^2 \).

   (a) State the domain of \( g \).

   (b) Determine if \( g \) has any asymptotes; your reasoning must utilize limits and determine the behavior of \( g \) at each end of the domain you found in (a).

   (c) On what interval(s) is \( g \) increasing? decreasing? Explain completely.
(d) On what interval(s) is $g$ concave up? concave down? Explain completely.

(e) Using the information you’ve found, sketch the graph $y = g(x)$. Label and provide the $(x, y)$ coordinates of all local extrema and inflection points.
7. (10 points) Compute the following limits, showing all reasoning.

(a) \[ \lim_{x \to 0} \frac{1 - \cos(x^2)}{x^4} \]

(b) \[ \lim_{x \to \infty} \left( \frac{1}{1 + 2x} \right)^{1/x} \]
8. (10 points) In the $xy$-plane, any negatively-sloped line that passes through the point $(2,3)$ will form a right triangle with the $x$- and $y$-axes in the first quadrant. Among all possible such lines (negative slope, passing through $(2,3)$), find the equation of the line that forms a triangle of minimal area. Justify completely.
9. (7 points) The equation $x \ln x = 1$ has a unique solution. Use Newton’s method with the initial guess $x_1 = 1$ to produce two successive approximations to the solution, namely $x_2$ and $x_3$. Show all of your steps.
10. (6 points) The acceleration, in m/sec$^2$, of a particle moving along a line is given as a function of time $t$ (in sec) by the formula

$$a(t) = 2e^t + 3 \sin t - t.$$ 

The initial velocity (at time $t = 0$) is 2 m/sec. What is the particle’s velocity at time $t = 4$?