Math 41: Calculus
First Exam — October 13, 2009

Name: ___________________________  SUID#: ____________

Select your section:

<table>
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<th>Section</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atoshi Chowdhury</td>
<td>02 (11-11:50am)</td>
</tr>
<tr>
<td>Yuncheng Lin</td>
<td>08 (10-10:50am)</td>
</tr>
<tr>
<td>Ian Petrow</td>
<td>04 (1:15-2:05pm)</td>
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<tr>
<td>Ha Pham</td>
<td>03 (11-11:50am)</td>
</tr>
<tr>
<td>Yu-jong Tzeng</td>
<td>41A</td>
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</tbody>
</table>

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.

- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.

- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.

- Please check that your copy of this exam contains 11 numbered pages and is correctly stapled.

- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, October 27**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

- Please sign the following:

  “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

  **Signature: ___________________________**

The following boxes are strictly for grading purposes. Please do not mark.

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<th>13</th>
<th>6</th>
<th>10</th>
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<tbody>
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<td>2</td>
<td>20</td>
<td>7</td>
<td>16</td>
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<td>3</td>
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<td>5</td>
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<td>Total</td>
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1. (13 points) Find each of the following limits, with justification. If the limit does not exist, explain why. If there is an infinite limit, then explain whether it is $\infty$ or $-\infty$.

(a) \[ \lim_{x \to 2} \frac{x^2 - 4}{x^2 + x - 2} \]

(b) \[ \lim_{x \to -2^+} \frac{5x + 10}{x^3 + 4x^2 + 4x} \]
(c) \( \lim_{x \to 3} (f \circ g)(x) \), where \( f(x) = \sin(\pi x) \) and \( g \) satisfies

\[
\begin{align*}
\lim_{x \to 3^+} g(x) &= 1 \quad \text{and} \quad \lim_{x \to 3^-} g(x) = 5.
\end{align*}
\]
2. (20 points) Consider the function
\[
f(x) = \begin{cases} 
\sin x & \text{if } x < \pi \\
\frac{\pi}{x} \sin x & \text{if } x \geq \pi 
\end{cases}
\]

For each of the statements below, circle whether it is TRUE or FALSE. To receive credit, please also explain your reasoning completely.

(a) \( f(x) \) is a one-to-one function. TRUE FALSE

(b) The graph of \( y = f(x) \) has a vertical asymptote. TRUE FALSE

(c) \( f(x) \) is continuous at \( x = \pi \). TRUE FALSE
For quick reference, here again is the definition of $f$:

$$f(x) = \begin{cases} 
\sin x & \text{if } x < \pi \\
\frac{\pi}{x} \sin x & \text{if } x \geq \pi
\end{cases}$$

(d) $f'(x)$ (i.e., the derivative of $f$) has a horizontal asymptote.  TRUE  FALSE

(e) $f''(x) > 0$ whenever $\pi < x < 2\pi$.  TRUE  FALSE
3. (8 points) Let \( f(x) = \sqrt{x^2 + x + 1} \). Find a formula for \( f'(x) \) using the limit definition of the derivative. Show the steps of your computation.
4. (8 points)

(a) Is the function \( h(x) = \frac{1}{x + 2} \) continuous on its domain? Explain your answer.

(b) Suppose the function \( g \) satisfies \( |g'(x)| \leq 4 \) for every value of \( x \). Must \( g \) be continuous at every value of \( x \)? If so, explain why this is the case; if not, give a counterexample showing why not.
5. (6 points) Using the graph below of $y = f(x)$, list the following quantities in increasing order (from smallest to largest). No justification is necessary.

\[ f'(2) \quad f'(4) \quad 0 \quad f''(2) \quad f(3) - f(2) \quad \frac{1}{2}[f(4) - f(2)] \]
6. (10 points) The period $P$ of a pendulum (time for one full swing and back), measured in seconds, is observed to be a function of the length $x$ of the pendulum, measured in inches, according to the following chart:

<table>
<thead>
<tr>
<th>$x$</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>0.64</td>
<td>0.78</td>
<td>0.90</td>
<td>1.01</td>
<td>1.11</td>
<td>1.20</td>
</tr>
</tbody>
</table>

(a) Estimate the value of $P'(10)$. What are its units?

(b) What is the practical meaning of the quantity $P'(10)$? Give a brief but specific one- or two-sentence explanation that is understandable to someone who is not familiar with calculus.

(c) The ideal clock pendulum requires a period of exactly one second. According to the available information, is an ideal clock pendulum shorter or longer than 9.9 inches? Explain your answer.
7. (16 points) Find the derivative, using any method you like. You do not need to simplify your answers.

(a) \( p(x) = x^7 + x\sqrt{7} - \frac{\sqrt{2}}{1 + \pi} \)

(b) \( r(t) = \frac{\sin t}{1 + \cos t} - \frac{1 + \cos t}{\sin t} \)

(c) \( h(z) = \frac{z^2 e^z}{15} + \frac{1}{z\sqrt{z}} \)

(d) \( L(t) = (1 + t + 2t^2)(t \sin t - \cos t) \)
8. (6 points) Find (with justification) values of constants $a$, $b$, and $c$ so that the functions

$$f(x) = \cos x$$

and

$$g(x) = a + bx + cx^2$$

simultaneously satisfy all of the following conditions:

$$f(0) = g(0), \quad f'(0) = g'(0), \quad f''(0) = g''(0).$$
9. (13 points) Sketch the graph of a function \( g \) with all of the following properties. Be sure to label the scales on your axes.

- The domain of \( g \) is \(( -\infty, \infty )\)
- \( g \) is continuous everywhere except at \( x = 3 \) (where \( g \) is not continuous)
- \( g(3) = 0 \)
- \( \lim_{x \to -\infty} g(x) = -\infty \)
- \( \lim_{x \to \infty} g(x) = 1 \)
- Both \( \lim_{x \to 3} g(x) \) and \( \lim_{x \to 3} g'(x) \) exist
- \( g'(x) < 0 \) if \( x > 3 \) or \( 2 < |x| < 3 \)
- \( g'(x) > 0 \) if \( x < -3 \) or \( |x| < 2 \)
- \( \lim_{x \to -3} |g'(x)| = \infty \)
- \( g''(x) < 0 \) if \( 0 < x < 3 \)
- \( g''(x) > 0 \) if \( -3 < x < 0 \) or \( |x| > 3 \)