• Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.

• You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.

• **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.

• Please check that your copy of this exam contains 12 numbered pages and is correctly stapled.

• If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

• It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, October 26**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

• Please sign the following:

  “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

  **Signature**: __________________________

The following boxes are strictly for grading purposes. Please do not mark.

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<th>Question</th>
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1. (13 points) Find each of the following limits, with justification. If the limit does not exist, explain why. If there is an infinite limit, then explain whether it is $\infty$ or $-\infty$.

(a) $\lim_{x \to 5^-} \left( \frac{x - 5}{x} \cdot \frac{e^x}{x^2 - 6x + 5} \right)$

(b) $\lim_{x \to -\infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$
(c) \( \lim_{x \to 0^+} \sqrt{x} \cos \left( \sin \frac{1}{x} \right) \)
2. (15 points) Let \( f(x) = \frac{4e^x + 1}{2e^x - 3} \).

(a) Find the equations of all \emph{vertical} asymptotes of \( f \), or explain why none exist. As justification for each asymptote \( x = a \), compute at least one of the one-sided limits \( \lim_{x \to a^+} f(x) \) or \( \lim_{x \to a^-} f(x) \), showing your reasoning.

(b) Find the equations of all \emph{horizontal} asymptotes of \( f \), or explain why none exist. Justify using limit computations.
For easy reference, recall that $f(x) = \frac{4e^x + 1}{2e^x - 3}$.

(c) It is a fact that $f$ is a one-to-one function. Find an expression for $f^{-1}(x)$, the inverse of $f$. 
3. (8 points) Find values of $a$ and $b$ so that the function

$$f(x) = \begin{cases} \frac{a}{x-1} + 3 & \text{if } x < 2 \\ bx^2 - 4x & \text{if } x \geq 2 \end{cases}$$

is continuous on $(-\infty, \infty)$. Justify your answer.
4. (8 points) Let \( f(x) = \frac{x - 5}{x - 4} \). Find a formula for \( f'(x) \) using the limit definition of the derivative. Show the steps of your computation.
5. (6 points) Explain completely whether $f(x) = \sin(|x|)$ is differentiable at $x = 0$, and if it is, find $f'(0)$. Your reasoning should be complete; in particular, a picture alone is not sufficient.
6. (12 points) Data taken from a certain speed camera outside Santa Fe, NM, shows that traffic speed $S$, in mph, varies as a function of the traffic density $q$ (in cars per mile) at that point on the road. The following table gives $S$ for a few values of $q$:

<table>
<thead>
<tr>
<th>$q$</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(q)$</td>
<td>46</td>
<td>43</td>
<td>40</td>
<td>38</td>
<td>37</td>
</tr>
</tbody>
</table>

(a) Estimate the value of $S'(120)$, and give its units.

(b) What is the practical meaning of the quantity $S'(120)$? Give a brief but complete one- or two-sentence explanation that is understandable to someone who is not familiar with calculus.

(c) The product $V(q) = q \cdot S(q)$ is called traffic volume. What are its units? What is its practical meaning?

(d) Find a formula for $V'(q)$. (Express your answer in terms of other quantities, such as $q$, $S(q)$, and $S'(q)$.)
7. (6 points) The graph of the function $g$ is given below, as well as the graphs of the function’s first and second derivatives, $g'$ and $g''$, respectively. Indicate which graph belongs to which function, and give your reasoning.
8. (12 points) Find the derivative, using any method you like. You do not need to simplify your answers.

(a) \( R(t) = t^{-7.3} + \frac{t^4 + \sqrt{t}}{t} \)

(b) \( f(x) = xe^x - e^x + 3\pi^2 x^7 \)

(c) \( P(y) = \frac{\cos y - 1}{y \sin y} \)
9. (15 points) The following is a graph of the function $g$:

(a) On the axes below, sketch a plausible graph of $g'$, the derivative of the function graphed above. Label the scales on your axes.
New information: Now suppose instead that we think of the above as the graph of the derivative of some function $h$. Give brief answers to the following questions about the function $h$. You do not need to provide justification for your answers. (But be careful: the questions below pertain to $h$, even though we are now supposing the above graph is a picture of $h'$, the derivative.)

(b) On what intervals is $h$ increasing? decreasing?

(c) On what intervals is $h$ concave up? concave down?

(d) Now suppose additionally that that $h(0) = 0$ and $h$ is continuous on $(-\infty, \infty)$. Sketch a plausible graph of $h$ on the set of axes below. Label the vertical scale you use, and clearly label the locations of horizontal tangents ("HT") and points of inflection ("IP").
(This page intentionally left blank for scratch work.)