Mathematics Department Stanford University
Survival Tips for Math 51H

1. Overall approach to the class:

Most days in the Math 51H class you will be presented with new material, usually at a pace which makes it difficult to fully appreciate the subtleties and significance of the various definitions and results. For this reason it is essential that you take accurate notes of what has been covered, so that you can make a detailed review later (as discussed in point 2 below). In an honors class it is usually not the case that the lecturer simply "follows the text," so the taking of accurate notes for later review becomes doubly important.

Try to keep in mind at all times that the various results and concepts should all fit together as an integrated whole which ultimately makes sense and which seems down-to-earth and logical.

If your level of assimilation of the material reaches a point where this is the case, then you will find that very little "memorization" and "rote learning" are involved; such a state of mathematical maturity can only be attained through consistent effort and constant review of the material. Helping you to achieve such a state is one of the main aims of Math 51H. Try not to be discouraged too quickly if you do not feel you are achieving this immediately—many students with outstanding mathematical potential have trouble in the first stages. This may be partly explained by the fact that many have an experience in high school where rote learning and routine application of formulae are almost exclusively what is emphasized; in many cases students, even those with excellent mathematical potential, have never developed the skill of being able to sit down and think about a problem which initially seems to be quite new to them and is not simply a rehash of a "problem" which you have seen a number of times before in just a slightly different guise.

Keep in mind also that in Math 51H all results will be proved—virtually nothing is to be taken on faith except for the basic axioms of set theory and the real numbers. So you should never be in a situation of using results which you do not know how to prove rigorously.

2. Reviewing Lecture Material:

After each lecture (the same day or early the next morning) you should ask yourself at least the following questions: What were the main 4 or 5 points covered in today's lecture? Can I at least state those results? To what extent do I already understand these results? For example suppose you look at the definition of "subspace of $\mathbb{R}^n$" (as on p.5 of the text) and "span" as on p.6 of the text. The natural tendency is simply to read those definitions and to say "yes I certainly understand that." But you have to accept that "understanding" applies at various levels; for instance can you easily and directly apply the definition of subspace and span to prove that if you are given vectors $\underline{v}_1, \ldots, \underline{v}_k \in \mathbb{R}^n$ then the span of $\underline{v}_1, \ldots, \underline{v}_k$ is a subspace of $\mathbb{R}^n$. If you can't do that more or less immediately then it is fair to say that you certainly do not "understand" the concepts of "subspace" and "span," at least not at a level where those concepts become useful. The same principle applies to just about every topic we cover in 51H—you really have to develop the ability to seriously ask yourself the question "do I understand that?"

As part of any review of basic material (either one or several lectures' worth of material) you should examine special or "extreme" cases of definitions and theorems, as well as standard examples. Also try to see if there are unusual or non-obvious cases of the result which have important consequences or which tie it to results or concepts covered previously. Remember, a primary overall aim is to be able to view the totality of the material as an integrated whole, with many parts which fit together in a clear and elegant way, so pay particular attention to how the different parts of the course fit together. Try to construct some sort of chart which graphically represents how the different results, and their proofs, are connected, and keep this under constant review. You'll be surprised how useful, and at the same time complex, this can be.

3. Homework:

The weekly homework is of paramount importance. It is the main way in which you come to terms with the material presented in the the lectures, and start to really assimilate it, "understand" it in at least the sense discussed in point 2 above, and apply it to problems which are at least one level removed from direct "rote" application. You'll notice that only 15% of the total score for the course is allocated for homework; that is done not because the homework is of lesser importance than the tests (on the contrary it is the most important learning tool provided to you), but rather to encourage you to tackle the homework with more concern about using it to build your understanding and less worry about getting a good score. Clearly if you get too much help, give up and ask for help before you have even seriously thought about what could be involved in the solution of the problem at hand, then you might get a good score on the homework but have gained very little in real understanding of the material. Of course if you are completely stuck on one of the problems and even after thinking seriously about it you are unable to get a start, then you need to ask for help. But even in those circumstances you should not make the mistake of asking the TA "how do I do this problem?" Rather you should ask for a hint as to what is involved in getting the problem started, and then go on from there. If you later find you are still stuck then of course you can again ask for help; but do keep in mind that real mathematical ability is generally only developed by struggling with the various problems set in the course, not by imitating someone else's solution to those problems.

Each week you should review the homework (and all test problems) after they have been graded and returned to you. With the aid of the solution set, actively work out where your problems were and what is still not clear. Keep an ongoing list of things that are causing difficulties, and review it regularly. Try to analyze this list to see if you can identify the basic difficulties from which all other problems/confusion arise. This practice (keeping track of your difficulties and analyzing them as thoroughly as possible) is a very important part of the mathematical learning process.

4. Reviewing for Tests and Exams:

Make sure you review all the main results and theorems that have been covered in class and in homework. At minimum you should be able to fluently provide the detailed statements of all theorems and results covered, as well as giving the proofs, which includes the ability to identify and analyze the main points involved in those proofs, including special cases and important

applications. This is not something you are going to achieve in a night's study before the test; indeed it will only be possible if you have consistently worked on the material and built up your understanding in stages. So don't leave your test preparation until the last minute. Unless you've put in the hard hours earlier that will not work.

5. What to do if things seems to be spinning out of control:

When you fall behind and comprehension fails, it seems like you have a multitude of difficult concepts to juggle and things seem to be getting more and more confused. What starts out as one or two points of confusion can rapidly degenerate into a state in which even the things that you thought you understood earlier are now unclear and confusing. As the stress caused by this situation rises so your ability to think clearly diminishes. It really is a bad place to be. If you feel you are in a situation like this, or at least are heading in that direction, and you don't want to drop the class, then you have to accept that a radical change in approach is called for—don't delude yourself that the class will come out OK if you just keep going as you are. You must immediately begin a radical review program to identify and weed out all the areas which are causing you difficulty. That will be time-consuming and will probably require some outside help from TA and fellow students, but of course the real work will have to be done by yourself—outside help can only be of limited value when you are coming to grips with the essential material.