Final Exam
ME111 Stress, Strain and Strength
Monday, December 6, 1999
Three hours
Open Notes and Textbook (Only)

SOLUTION

Problem 1 (20 points)

A machined shaft is subjected to a fully reversed torque of 8.5 in-klb. Determine the diameter of the shaft needed to provide a factor of safety of 2.5 if a life of 10^5 cycles is required. The material properties are:

$$S_{ut} = 220\,kpsi, \quad S_y = 67\,kpsi,$$
$$S_m = 0.9 S_{ut}, \quad S_e = 100\,kpsi \quad (fully\,corrected)$$

$$\tau_a = \frac{Tc}{I} = \frac{8.5 * d/2}{\pi d^4/32} = 43.29\frac{1}{d^3}$$
$$\sigma' = \sqrt{3\tau_a^2} = \tau_a\sqrt{3} = \frac{74.98}{d^3}$$
$$Now\,N_f = 2.5 = \frac{S_f}{\sigma'}$$
$$Compute\,S_f:$$
$$S_f = aN^b$$
$$b = -\frac{1}{3}\log_{10}\left(\frac{S_m}{S_e}\right) = -\frac{1}{3}\log_{10}\left(\frac{0.9S_{ut}}{S_e}\right) = -0.09889$$
$$\log_{10}(a) = \log_{10}(S_m) - 3b = \log_{10}(0.9S_{ut}) - 3b = 2.5933 \Rightarrow a = 392.04$$
$$S_f = 392.4 * 100{,}000^{-0.09889} = 125.57$$
$$Now,$$
$$\sigma' = \frac{S_f}{2.5}; \quad \sigma' = \frac{74.98}{d^3}; \quad S_f = 125.57$$
$$\Rightarrow d^3 = \frac{74.98 * 2.5}{125.57} = 1.493; \qquad d = 1.14\,in$$

Problem 2 (20 points)

In a machine component subject to a fluctuating multiaxial stress state, the von Mises effective alternating and mean stress are:

$$\sigma_a' = 24\,kpsi, \quad \sigma_m' = 40\,kpsi$$

The relevant material properties are:

$$S_{ut} = 80\,kpsi, \quad S_y = 67\,kpsi,$$
$$S_m = 0.9S_{ut}, \quad S_e = 40\,kpsi \quad (\,fully\ corrected)$$

(a) Determine whether or not the element is safe for infinite life fatigue (hint: compute the appropriate factor of safety assuming a fixed ratio of stresses),

(b) If it is not safe for infinite life, estimate the number of cycles to failure.

Part (a)

$$For\ infinite\ life\ fatigue:$$

$$N_f = \frac{S_e S_{ut}}{\sigma_a' S_{ut} + \sigma_m' S_e} = 0.909 \Rightarrow not\ safe\ for\ infinite\ life\ fatigue.$$

$$Note: no\ failure\ by\ yielding:\ N_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{67}{24+40} = 1.05$$

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Part (b)

$$For\ failure\ in\ HCF:$$

$$N_f = 1 = \frac{S_f S_{ut}}{\sigma_a' S_{ut} + \sigma_m' S_f} \Rightarrow S_f = 48.0$$

$$Now\ S_f = aN^b$$

$$b = -\frac{1}{3}\log_{10}\left(\frac{S_m}{S_e}\right) = -\frac{1}{3}\log_{10}\left(\frac{0.9S_{ut}}{S_e}\right) = -0.085$$

$$\log_{10}(a) = \log_{10}(S_m) - 3b = \log_{10}(0.9S_{ut}) - 3b = 2.1126$$

$$a = 129.6$$

$$Giving: \quad 48.0 = 129.6N^{-0.085} \Rightarrow N = 117{,}341\ cycles$$

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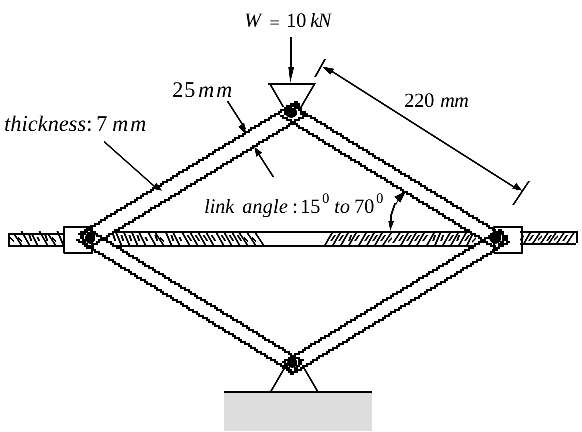
Problem 3 (20 points)

The figure below shows a schematic drawing of an automobile jack which is subject to a load of W = 10 kN. The opposite-handed threads on the two ends of the screw are cut to allow the link angle to vary from 15º to 70º. The links are rectangular in cross-section with dimensions 25 x 7 mm. They are made from steel with E = 206.8 GPa and S_y = 290 MPa. If the load is to be supported over the full range of the jack, what is the factor of safety against buckling of the links? Assume that the axial load in the links is central (i.e. there is no eccentricity) and that the links can be modeled as pinned-pinned about both axes.



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Compute axial load in link from statics:

$$2 * P_{link} * \sin 15^0 = W \Rightarrow P_{link} = \frac{W}{2 * \sin 15^0} = 19.318 kN$$

$$\rho = \sqrt{\frac{bd^3/12}{bd}} = \frac{d}{\sqrt{12}} = \frac{7}{\sqrt{12}} = 2.02 \quad (using\ weak\ axis)$$

$$S_r = \frac{L}{\rho} = \frac{220}{2.02} = 108.87$$

$$(S_r)_{y/2} = \pi\sqrt{\frac{2E}{S_y}} = 118.6 \Rightarrow Johnson\ column$$

$$\frac{P_{cr}}{b * d} = S_y - \frac{1}{E}\left(\frac{S_y S_r}{2\pi}\right)^2 \Rightarrow P_{cr} = 29.382 kN$$

$$N_b = \frac{P_{cr}}{P_{link}} = \frac{29.382}{19.318} = 1.52$$

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Problem 4 (20 points)

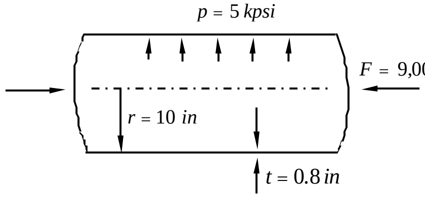
A thin-walled cylindrical pressure vessel has an inner radius of r = 10 in. and a wall thickness of t = 0.8 in. The pressure vessel supports an internal pressure of p = 5 kpsi and an axial compressive load of N = 9000 klb.

Consider the following:

(a) The pressure vessel is made of a ductile material with a yield strength of S_y = 250 kpsi. Determine the factor of safety against yielding under the applied load using:

(i) The von Mises (i.e. distortion energy) criterion,
(ii) The maximum shear stress criterion.

(b) The material is brittle with an ultimate tensile strength of S_{ut} = 180 kpsi. What is the minimum ultimate compressive stress S_{uc} that is required of the material to provide a factor of safety of 1.5 against brittle failure? Use the modified Mohr theory and assume plane stress conditions.



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Part (a)

$$\sigma_1 = \frac{pr}{t} = 62.5$$

$$\sigma_2 = -p = -5.0$$

$$\sigma_3 = \frac{pr}{2t} - \frac{P}{A} = -140.9$$

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1} = 179.45$$

$$von\,Mises: \quad N_y = \frac{S_y}{\sigma'} = \frac{250}{179.45} = 1.39$$

$$Tresca: \quad N_y = \frac{S_y}{\max(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|)} = \frac{S_y}{|\sigma_3 - \sigma_1|} = \frac{250}{203.4} = 1.23$$

Part (b)

$$Since\ fourth\ quadrant\ and\ |\sigma_3| > |\sigma_1|,$$

$$N_b = 1.5 = \frac{S_{ut}S_{uc}}{S_{uc}\sigma_1 - S_{ut}(\sigma_1 + \sigma_3)}$$

$$or\ \frac{180 * S_{uc}}{S_{uc} * 62.5 - 180(62.5 - 140.9)} = 1.5 \Rightarrow S_{uc} = 245.5\ kpsi$$

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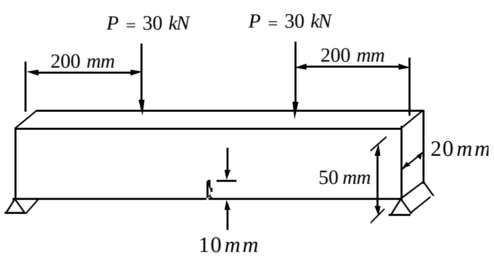
Problem 5 (20 points)

The beam shown has depth of *50* mm and a thickness of *20 mm*. A crack through the thickness of the beam is detected in the bottom face as indicated. The crack is measured to be *10 mm* long and it needs to be determined if the beam can safely support the two point loads *P = 30 kN*. The beam is fabricated from AISI 4340 steel with a yield strength $S_y = 800\ MPa$ and a fracture toughness $K_c = 185\ MPa(m)^{1/2}$.

(a) Using LEFM, determine the value of the load P that will cause the crack to propagate.

(b) Repeat (a) taking plasticity at the crack tip into account.

(c) Can the beam support the design load of P = 30 kN?



Part (a)

$$\frac{a}{d} = 0.2 \Rightarrow \beta \approx 1.0$$

$$K = K_C = 185 = \beta\sigma\sqrt{\pi a}; \qquad \sigma = \frac{Mc}{I} = \frac{200P*50/2}{20*50^3/12}$$

$$Now \quad 185 = (1.0)\left(\frac{200P*25}{20*50^3/12}\right)\sqrt{\pi\frac{10}{1000}} \Rightarrow P = 43.5\,kN$$

Part (b)

$$K = K_C \Rightarrow a_{eff} = a + \underbrace{\frac{1}{\pi}\left(\frac{K_C}{S_y}\right)^2}_{meters} * 1000 = 27.0\,mm$$

$$\frac{a_{eff}}{d} = 0.54 \Rightarrow \beta \approx 1.7$$

$$K = K_C = 185 = \beta\sigma\sqrt{\pi a_{eff}}$$

$$Now \quad 185 = (1.7)\left(\frac{200P*25}{20*50^3/12}\right)\sqrt{\pi\frac{27}{1000}} \Rightarrow P = 15.6\,kN$$

Part (c)

No, fails when plasticity at the crack tip is considered.