A machined shaft is subjected to a fully reversed torque of 8.5 in-klb. Determine the diameter of the shaft needed to provide a factor of safety of 2.5 if a life of $10^5$ cycles is required. The material properties are:

- $S_u = 220 \text{ kpsi}$
- $S_s = 67 \text{ kpsi}$
- $S_m = 0.95 S_u$
- $S_y = 100 \text{ kpsi}$ (fully corrected)

Problem 1 (20 points)

SOLUTION

**Compute $S_f$:**

$$b = -\frac{1}{3} \log \left( \frac{S_n}{S_y} \right) = -\frac{1}{3} \log \left( \frac{0.95 S_u}{S_y} \right) = -0.09889$$

$$\log_{10}(a) = \log_{10}(S_n) - 3b = \log_{10}(0.95 S_u) - 3b = 2.5933 \Rightarrow a = 39204$$

$$S_f = 3924 + 10000 \times 0.09889 = 12557$$

Now,

$$\sigma' = \frac{S_f}{2.5} \Rightarrow \sigma' = \frac{74.98}{2.5} \Rightarrow S_f = 12557$$

$$\Rightarrow d = \frac{74.98 \times 2.5}{12557} = 1.493 \Rightarrow d = 1.14 \text{ in}$$
In a machine component subject to a fluctuating multiaxial stress state, the von Mises effective alternating and mean stress are:

\[ \sigma_a = 24 \text{kpsi}, \quad \sigma_m = 40 \text{kpsi} \]

The relevant material properties are:

\[ S_u = 80 \text{kpsi}, \quad S_y = 67 \text{kpsi}, \]
\[ S'_u = 0.9S_u, \quad S_y = 40 \text{kpsi} \quad \text{(fully corrected)} \]

(a) Determine whether or not the element is safe for infinite life fatigue (hint: compute the appropriate factor of safety assuming a fixed ratio of stresses).

(b) If it is not safe for infinite life, estimate the number of cycles to failure.

Part (a)

For infinite life fatigue:

\[ N_f = \frac{S_u S_y}{\sigma_a S_u + \sigma_m S_y} = 0.909 \Rightarrow \text{not safe for infinite life fatigue.} \]

Note: no failure by yielding: \[ N_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{67}{24 + 40} = 1.05 \]

Part (b)

For failure in HCF:

\[ N_f = 1 - \frac{S_u S_y}{\sigma_a S_u + \sigma_m S_y} \Rightarrow S_f = 48.0 \]

Now \[ S_f = aN^b \]

\[ b = \frac{1}{3} \log_{10} \left( \frac{S_y}{S_u} \right) = \frac{1}{3} \log_{10} \left( \frac{0.95 S_u}{S_y} \right) = -0.085 \]
\[ \log_{10}(a) = \log_{10}(S_y) - 3b = \log_{10}(0.95 S_u) - 3 	imes 2.1126 = 1296 \]

Giving: \[ 48.0 = 1296N^{-0.085} \Rightarrow N = 117341 \text{cycles} \]
**Problem 3 (20 points)**

The figure below shows a schematic drawing of an automobile jack which is subject to a load of \( W = 10 \text{ kN} \). The opposite-handed threads on the two ends of the screw are cut to allow the link angle to vary from \( 15^\circ \) to \( 70^\circ \). The links are rectangular in cross-section with dimensions 25 x 7 mm. They are made from steel with \( E = 206.8 \text{ GPa} \) and \( S_y = 290 \text{ MPa} \). If the load is to be supported over the full range of the jack, what is the factor of safety against buckling of the links? Assume that the axial load in the links is central (i.e. there is no eccentricity) and that the links can be modeled as pinned-pinned about both axes.

\[ W = 10 \text{ kN} \]

25 mm

220 mm

link angle: \( 15^\circ \) to \( 70^\circ \)

**Compute axial load in link from statics:**

\[ 2 \cdot P_{\text{load}} \cdot \sin 15^\circ = W \Rightarrow P_{\text{link}} = \frac{W}{2 \cdot \sin 15^\circ} = 19.318 \text{ kN} \]

\[ \rho = \sqrt{\frac{bd^2/12}{bd}} = \frac{d}{\sqrt{12}} = \frac{7}{\sqrt{12}} = 2.02 \quad \text{(using weak axis)} \]

\[ S_y = \frac{L}{\rho} = \frac{220}{202} = 108.87 \]

\[ (S_y)_{1/2} = \pi \frac{S_y}{\sqrt{E}} = 118.6 \Rightarrow \text{Johnson's Column} \]

\[ P_{\text{cr}} = \frac{S_y}{b \cdot d} \cdot \left( \frac{S_y}{E \cdot 2\pi} \right) \Rightarrow P_{\text{cr}} = 29.382 \text{ kN} \]

\[ N_y = \frac{P_{\text{cr}}}{P_{\text{link}}} = \frac{29.382}{19.318} = 1.52 \]
Problem 4 (20 points)

A thin-walled cylindrical pressure vessel has an inner radius of \( r = 10 \text{ in.} \)
and a wall thickness of \( t = 0.8 \text{ in.} \). The pressure vessel supports an internal pressure of \( p = 5 \text{ kpsi} \) and an axial compressive load of \( N = 9000 \text{ klb} \).

Consider the following:

(a) The pressure vessel is made of a ductile material with a yield strength of \( S_y = 250 \text{ kpsi} \). Determine the factor of safety against yielding under the applied load using:
   (i) The von Mises (i.e. distortion energy) criterion,
   (ii) The maximum shear stress criterion.

(b) The material is brittle with an ultimate tensile strength of \( S_u = 180 \text{ kpsi} \). What is the minimum ultimate compressive stress \( S_{uc} \) that is required of the material to provide a factor of safety of 1.5 against brittle failure? Use the modified Mohr theory and assume plane stress conditions.

Part (a)

\[
\sigma_1 = \frac{P}{A} = 62.5
\]
\[
\sigma_2 = -p = -5.0
\]
\[
\sigma_3 = \frac{P}{2t} = -1409
\]
\[
\sigma' = \sqrt{\frac{1}{2} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \sigma_2 - \sigma_2 \sigma_3 - \sigma_3 \sigma_1)} = 179.45
\]
\[
von\text{Mises: } N_y = \frac{S_y}{\sigma'} = \frac{250}{179.45} = 1.39
\]
\[
Tresca: N_y = \max\left(\frac{S_y}{\sigma_1 - \sigma_3}, \frac{S_y}{\sigma_2 - \sigma_3}, \frac{S_y}{\sigma_2 - \sigma_1} \right) = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{S_y}{\sigma_2 - \sigma_1} = \frac{250}{203.4} = 1.23
\]

Part (b)

Since fourth quadrant\( \Rightarrow \frac{N_y}{S_y} \)
\[
N_y = 1.5 = \frac{S_{uc} S_{uc}}{S_{uc} (S_u - S_{uc} (1 + \sigma_3))}
\]
or\[
\frac{180 S_{uc}}{S_{uc} + 625 - 180(625 - 1409)} = 1.5 \Rightarrow S_{uc} = 2455 \text{ kpsi}
\]
The beam shown has a depth of 50 mm and a thickness of 20 mm. A crack through the thickness of the beam is detected in the bottom face as indicated. The crack is measured to be 10 mm long and it needs to be determined if the beam can safely support the two point loads $P = 30$ kN. The beam is fabricated from AISI 4340 steel with a yield strength $S_y = 800$ MPa and a fracture toughness $K_c = 185$ MPa($m$)$^{1/2}$.

(a) Using LEFM, determine the value of the load $P$ that will cause the crack to propagate.

(b) Repeat (a) taking plasticity at the crack tip into account.

(c) Can the beam support the design load of $P = 30$ kN?

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Problem 5 (20 points)

The beam shown has a depth of 50 mm and a thickness of 20 mm. A crack through the thickness of the beam is detected in the bottom face as indicated. The crack is measured to be 10 mm long and it needs to be determined if the beam can safely support the two point loads $P = 30$ kN. The beam is fabricated from AISI 4340 steel with a yield strength $S_y = 800$ MPa and a fracture toughness $K_c = 185$ MPa($m$)$^{1/2}$.

(a) Using LEFM, determine the value of the load $P$ that will cause the crack to propagate.

(b) Repeat (a) taking plasticity at the crack tip into account.

(c) Can the beam support the design load of $P = 30$ kN?

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Part (a)

\[
\frac{a}{d} = 0.2 \Rightarrow \beta = 1.0
\]

\[
K = K_c = 185 = \beta \alpha \sqrt{\pi a}; \quad \sigma = \frac{Mc}{I} = \frac{200P + 50}{20 \times 50^{1/2}}
\]

Now \( 185 = (1.0) \left( \frac{200P + 25}{20 \times 50^{1/2}} \right) \Rightarrow P = 43.5 \text{kN} \)

Part (b)

\[
K = K_c \Rightarrow \alpha_{eff} = a + \frac{1}{\pi} \left( \frac{K_c}{S_y} \right) \Rightarrow 1000 = 27.0 \text{ mm}
\]

\[
\frac{\alpha_{eff}}{d} = 0.54 \Rightarrow \beta = 1.7
\]

\[
K = K_c \Rightarrow \beta \alpha \sqrt{\alpha_{eff}} \Rightarrow 185 = (1.7) \sqrt{27} \Rightarrow P = 15.6 \text{kN}
\]

Part (c)

No, fails when plasticity at the crack tip is considered.