$7.5\times10/9\times12$

ME111
Instructor: Peter Pinsky
Class #1
September 27, 2000

Today's Topics

- The importance of analysis for mechanical design
- Overview of ME111
- Review of forces, work and power
- Review of statics: free-body diagrams; internal forces; equilibrium

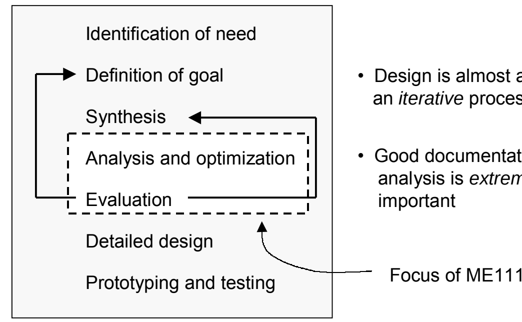
Reading Assignment Juvinall, Chapters 1 and 2.

Problem Set #1 Due in class 10/4/00

1. Juvinall 2.5
2. Juvinall 2.7
- Juvinall 2.10
- Juvinall 2.19

1.1 The Mechanical Design Process

- To design is to *formulate* a plan for the satisfaction of a (human) *need*



- Design is almost always an *iterative* process
- Good documentation of analysis is *extremely* important

Synthesis	"Puts together" the design idea
Analysis	Takes the design idea apart! Idealization (modeling) of elements, supports and loads Determination of critical points for stress and deflections Analysis depends on knowledge of *first principles*
Evaluation	Determination of level of safety against all possible *failure* modes

1.2 Evaluating Designs

- The machine or structure must be designed to operate with an adequate *factor of safety* against failure.
- Requires knowledge of all possible failure modes and the conditions that will cause them to be active.
- Examples of common failure modes include:

1. Fracture
2. Plastic (non-elastic) deformation
3. Excessive deflection
4. Buckling
6. Fatigue
7. Creep
8. Corrosion
9. Surface failure (e.g., fretting, wear)

Good design decisions require a good understanding of the behavior of materials

For example, the behavior of materials can be divided into two basic types:

DUCTILE “Stretchy” before failure	*BRITTLE* Relatively little stretch before sudden failure
low carbon steel, polymers, skin, rubber, aluminum	Cast iron, glass, ceramics, concrete, bone
exhibit “slow” failure with large deformations	exhibit “fast” explosive failure with small deformations

2.1 Forces

We will work with two systems of units:

	English Engineering	SI
Mass	lbm	kg
Length	ft	m
Time	s	s
Force	lb	N

Newtons second law

$$\mathbf{F}(lb) = m(lbm) * \frac{\mathbf{a}\left(ft \cdot s^{-2}\right)}{g_c} \qquad \mathbf{F}(N) = m(kg) * \mathbf{a}\left(m \cdot s^{-2}\right)$$

or

$$\mathbf{F}(lb) = \frac{m(lbm)}{g_c} * \mathbf{a}\left(ft \cdot s^{-2}\right)$$

Weight

Is the force required to accelerate unit mass in standard earth gravitational field

$$W(lb) = m(lbm) \qquad W(N) = m(kg) * 9.81$$

$$1\,lb = 4.448\,N \qquad 1\,N = 0.225\,lb$$

What is your weight in lb? In N?

What is the weight of a car in lb? In N?

Name an object that weights 1 N

2.2 Work and Power

Force -- displacement

$$W = \mathbf{F} \cdot \mathbf{S} \quad (ft \cdot lb;\ N \cdot m)$$

$$\dot{W} = \mathbf{F} \cdot \mathbf{V} \quad (hp;\ J)$$

Torque -- rotation

$$W = T \cdot \theta$$

$$\dot{W} = T \cdot \omega$$

$$1\, hp = 33{,}000\ \ ft \cdot lb / s$$

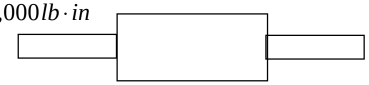
$$1\, watt(W) = N \cdot m / s$$

Example

A car transmission has a transmission ratio R=3.0 in first gear

$T_{engine} = 3{,}000\, lb \cdot in$

ω_{engine}



$T_{shaft} = ???$

ω_{shaft}

We know that

$$R = \frac{\omega_{engine}}{\omega_{shaft}} \Rightarrow \omega_{shaft} = \frac{\omega_{engine}}{R}$$

Assuming that power is not lost in the transmission

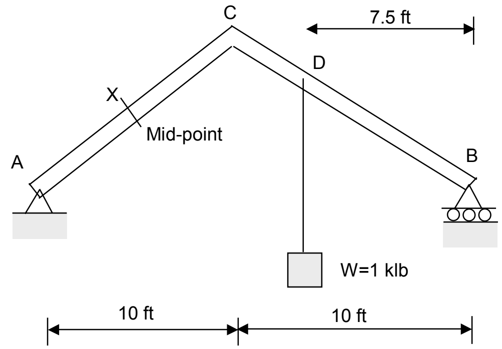
$$T_{engine} \cdot \omega_{engine} = T_{shaft} \cdot \omega_{shaft}$$

$$T_{shaft} = T_{engine} \cdot \frac{\omega_{engine}}{\omega_{shaft}} = T_{engine} \cdot R = 9{,}000\ lb \cdot in$$

3.1 Review of Statics -- Examples

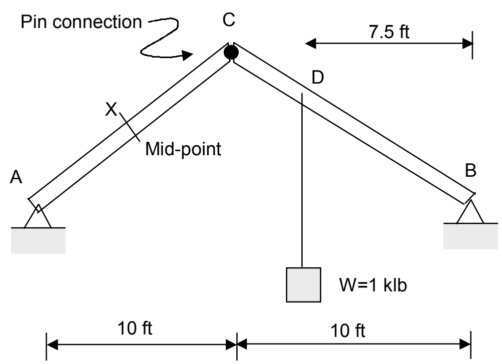
Example 1.1

For the *frame* shown, find (a) all support reactions, and (b) internal forces at point X.



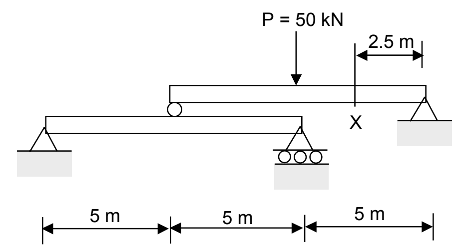
Example 1.2

For the *truss* shown, find (a) all support reactions, and (b) internal forces at point X.



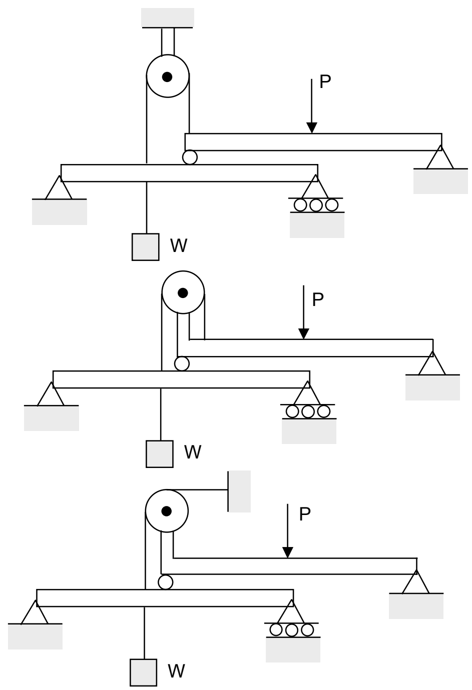
Example 1.3

Find (a) all support reactions, and (b) internal forces at point X.



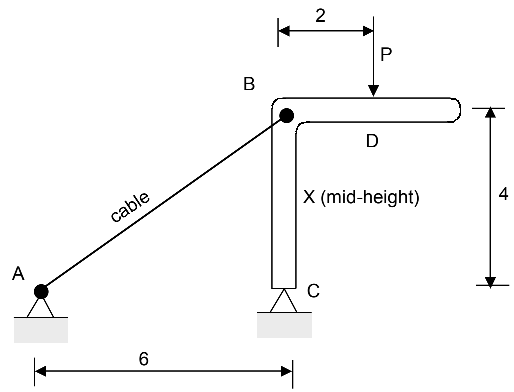
Example 1.4

Find (a) all support reactions, and (b) internal forces at point X.

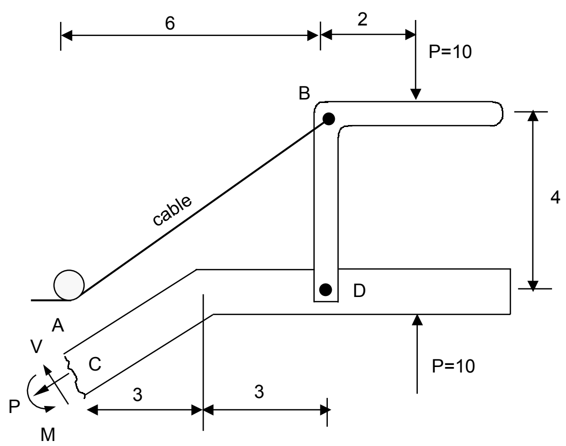


Example 1.5

Find (a) all support reactions, and (b) internal forces at point X.

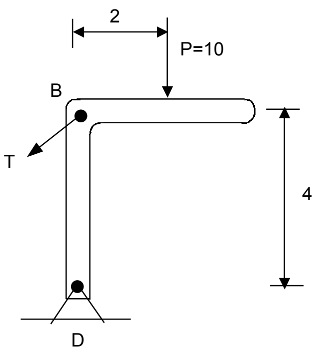


Example 1.6 (bicycle brake lever prototype)



Compute P, V, M
Is the handle bar in tension or compression?

Mathcad Solution



Compute tension in cable

$\theta := \operatorname{atan}\left(\frac{4}{6}\right)$ $T := 1$

Given

ΣM_D=0: $T \cdot \cos(\theta) \cdot 4 - 2 \cdot 10 = 0$

$T := \operatorname{Find}(T)$

$T = 6.009$

Compute internal forces

$P := 1$ $V := 1$ $M := 1$

Given

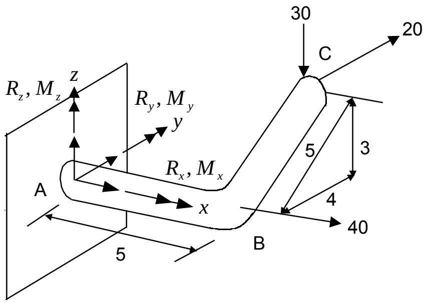
ΣF_x=0: $-T \cdot \cos(\theta) - P \cdot \cos(\theta) - V \cdot \sin(\theta) = 0$

ΣF_y=0: $-T \cdot \sin(\theta) - P \cdot \sin(\theta) + V \cdot \cos(\theta) = 0$

ΣM_D=0: $T \cdot \cos(\theta) \cdot 6 - T \cdot \sin(\theta) \cdot 6 + M = 0$

$$\operatorname{Find}(P, V, M) = \begin{pmatrix} -6.009 \\ 0 \\ -10 \end{pmatrix}$$

Example 1.7



3.2 Review of Statics -- Summary of equations

Statics in 2-d

Resultant of several forces

$$\mathbf{R} = \sum \mathbf{F} \qquad R = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$R_x = \sum F_x \quad R_y = \sum F_y \quad R_z = \sum F_z$$

Equilibrium of a particle

$$\mathbf{R} = \sum \mathbf{F} = \mathbf{0} \quad \text{or} \quad \sum \mathbf{F_x} = \mathbf{0}; \quad \sum \mathbf{F_y} = \mathbf{0}; \quad \sum \mathbf{F_z} = \mathbf{0}$$

Principle of transmissibility

The conditions of equilibrium or of motion of a rigid body are unchanged if a force acting on the body is moved along its line of action. Thus, forces acting on a rigid body may be treated as sliding vectors.

Vector product in 3-d:

$$\mathbf{V} = \mathbf{P} \times \mathbf{Q} \qquad V = PQ\sin\theta$$

$$\mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$

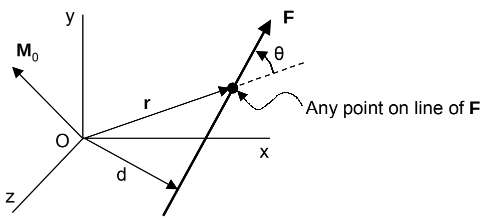
$$V_x = P_yQ_z - P_zQ_y$$
$$V_y = P_zQ_x - P_xQ_z$$
$$V_z = P_xQ_y - P_yQ_x$$

In 2-d:

$$\mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ P_x & P_y & 0 \\ Q_x & Q_y & 0 \end{vmatrix} = V_z\mathbf{k} \qquad V_z = P_xQ_y - P_yQ_x$$

Moment of a force about a point in three dimensions

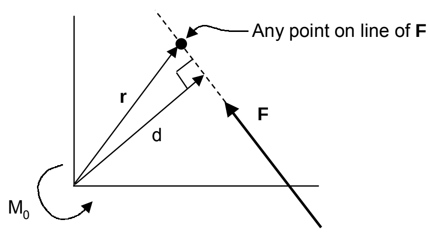
$\mathbf{M}_0 = \mathbf{r} \times \mathbf{F} \qquad M_0 = rF\sin\theta = Fd$



$$\mathbf{M}_0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$M_x = r_yF_z - r_zF_y$$
$$M_y = r_zF_x - r_xF_z$$
$$M_z = r_xF_y - r_yF_x$$

In 2-d:



$\mathbf{M}_0 = \mathbf{r} \times \mathbf{F} \qquad M_0 = rF\sin\theta = Fd$

$$\mathbf{M}_0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & 0 \\ F_x & F_y & 0 \end{vmatrix} = M_0\mathbf{k} \qquad M_0 = r_xF_y - r_yF_x$$

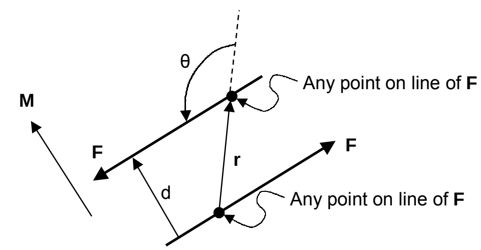
Varignon's theorem

The moment about any point of the resultant of several concurrent forces is equal to the sum of the moments of the individual forces about the same point.

Couples in 3-d

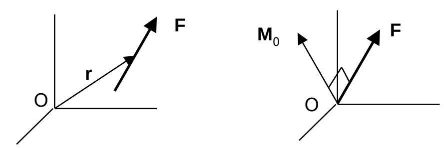
A couple consists of two parallel forces that are equal in magnitude and opposite in sense. The moment of a couple may be represented by a ***free*** vector

$\mathbf{M} = \mathbf{r} \times \mathbf{F} \quad M = rF\sin\theta = Fd$



Equivalent force-couple systems

A force may be tranlated parallel to itself to a new point of application provided a couple $\mathbf{M}_0 = \mathbf{r} \times \mathbf{F}$ is added.

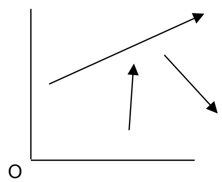


Equilibrium of a rigid body in 2-d

Three equations:

$\mathbf{R} = \sum \mathbf{F} = \mathbf{0}$ $\quad \sum F_x = 0 \quad \sum F_y = 0$

$\mathbf{M}_0^R = \sum \mathbf{M}_0 = \mathbf{0}$ $\quad \sum M_0 = 0$



Alternative sets of equations:

$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$ A = any point in x-y plane

$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$ AB not parallel to y-axis

$\sum M_A = 0 \quad \sum M_B = 0 \quad \sum M_C = 0$ A, B, and C not on straight line

Equilibrium of a two-force body

The two forces must be equal in magnitude, opposite in direction, and lie along same line.

Equilibrium of a three-force body

The three forces must lie in same plane and be either concurrent or parallel.

Equilibrium of a rigid body in 3-d

Six equations:

$\mathbf{R} = \sum \mathbf{F} = \mathbf{0}$ $\quad \sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$

$\mathbf{M}_0^R = \sum \mathbf{M}_0 = \mathbf{0}$ $\quad \sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$

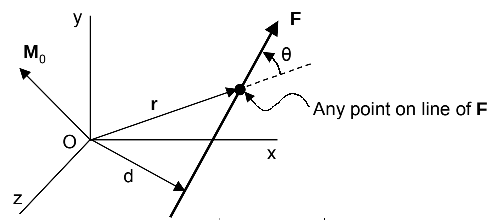
Statics in 3-d

Equilibrium of a rigid body in 3-d Six equations:

$\mathbf{R} = \sum \mathbf{F} = \mathbf{0}$ $\quad \sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$

$\mathbf{M}_0^R = \sum \mathbf{M}_0 = \mathbf{0}$ $\quad \sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$

Moment of a force about a point in three dimensions



$\mathbf{M}_0 = \mathbf{r} \times \mathbf{F}$

$M_0 = rF \sin\boldsymbol{\theta} = Fd$

$$\mathbf{M}_0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$M_x = \mathbf{M}_0 \cdot \mathbf{i} = \begin{vmatrix} r_y & r_z \\ F_y & F_z \end{vmatrix} = r_yF_z - r_zF_y = \text{Cross}\,(r_y, r_z, F_y, F_z)$$

$$M_y = \mathbf{M}_0 \cdot \mathbf{j} = -\begin{vmatrix} r_x & r_z \\ F_x & F_z \end{vmatrix} = \begin{vmatrix} r_z & r_x \\ F_z & F_x \end{vmatrix} = r_zF_x - r_xF_z = \text{Cross}\,(r_z, r_x, F_z, F_x)$$

$$M_z = \mathbf{M}_0 \cdot \mathbf{k} = \begin{vmatrix} r_x & r_y \\ F_x & F_y \end{vmatrix} = r_xF_y - r_yF_x = \text{Cross}\,(r_x, r_y, F_x, F_y)$$