ME111
Instructor: Peter Pinsky
Class #10
October 18, 2000

Today's Topic

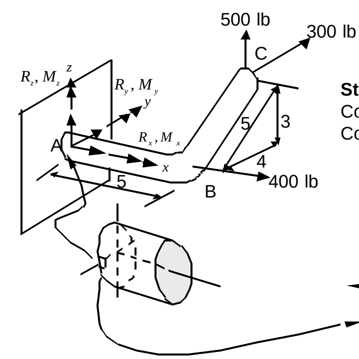
- Review of principal stresses in 3-d.
- Stresses in spherical and cylindrical pressure vessels.

Reading Assignment

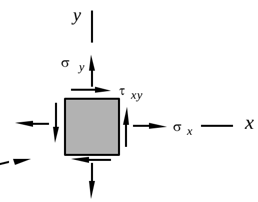
Problem Set #4 Due in class 10/25/00.

1. A spherical pressure vessel has a wall thickness of 2.5 mm and an Inner diameter of 150 mm, and it contains a liquid at 1.2 MPa pressure. Determine the maximum normal and shear stress.
2. A pipe 30 m long has closed ends, a wall thickness of 10 mm, and an inner diameter of 0.60 m. The pipe is filled with a gas to a pressure of 20 MPa, and is subjected to a torque about its long axis of 1.13 MN.m. ignoring end effects, determine the three principal normal stresses and the maximum shear stresses.
3. Consider an internally pressurized cylinder with with closed ends. Ignoring end effects, determine the principal normal and maximum shear stresses at the inner wall if the inner diameter is 80 mm, the outer diameter is 100 mm and the internal pressure is 100 MPa.

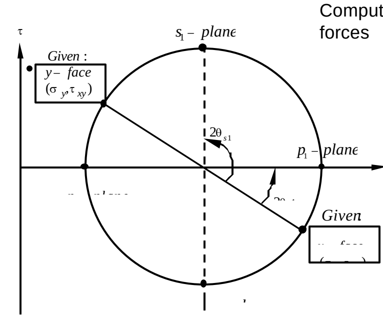
10.1 Review of Solution Process for Principal Stresses



Step 1
Compute reactions at support
Compute internal forces near A



Step 2
Compute stresses from internal forces



Step 3
Set up Mohr's circle
Determine principal stresses
Determine maximum shear stresses

10.2 Thin-Walled Pressure Vessels

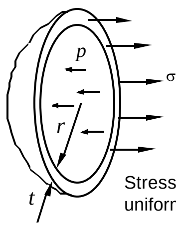
- Pressure vessels are closed structures that contain liquid or gas under pressure (e.g. water-storage tanks, compressed air containers, pressurized pipes).

- We consider first the special case of thin-walled pressure vessels:

$$\frac{r}{t} \geq 10$$

where r is the inner radius of the pressure vessel and t is its wall thickness

10.3 Thin-Walled Spherical Pressure Vessels



Stress assumed uniform over t

$$\sum F = 0: \quad \sigma\,(2\pi r_m t) - p\left(\pi r^2\right) = 0$$

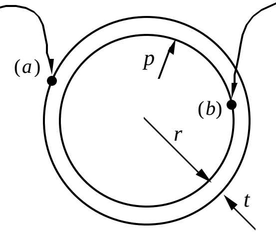
Taking $r_m \approx r$

$$\boxed{\sigma = \frac{pr}{2t}}$$

Called a membrane stress.

(a):

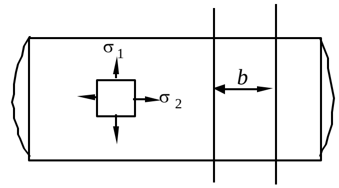
$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$
$$\sigma_3 = 0$$
$$\Rightarrow \tau_{max} = \frac{pr}{4t}$$

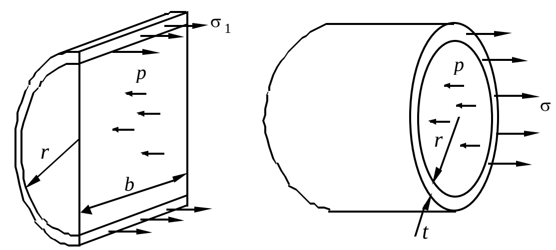


(b):

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$
$$\sigma_3 = -p$$
$$\Rightarrow \tau_{max} = \frac{pr}{4t} + \frac{p}{2}$$

10.4 Thin-Walled Cylindrical Pressure Vessels





$$\sum F = 0$$
$$\sigma_1(2bt) - p(2br) = 0$$
$$\sigma_1 = \frac{pr}{t}$$

$$\sum F = 0$$
$$\sigma_2(2\pi rt) - p\left(\pi r^2\right) = 0$$
$$\sigma_2 = \frac{pr}{2t}$$

On outer surface

$$\sigma_3 = 0 \qquad \tau_{max} = \frac{|\sigma_1 - \sigma_3|}{2} = \frac{pr}{2t}$$

On inner surface

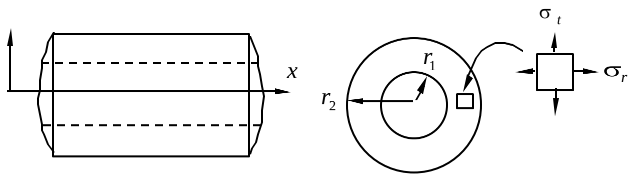
$$\sigma_3 = -p \qquad \tau_{max} = \frac{|\sigma_1 - \sigma_3|}{2} = \frac{\sigma_1 + p}{2} = \frac{pr}{2t} + \frac{p}{2}$$

10.4 Thick-Walled Cylindrical Pressure Vessels

• Thick-walled pressure vessels have:

$$\frac{r_1}{t} < 10$$

where r_1 is the inner radius of the pressure vessel and t is its wall thickness



• At a radius R, thick-walled pressure vessels have:

$$\sigma_t = -\frac{p r_1^2}{r_2^2 - r_1^2}\left(\frac{r_2^2}{R^2} + 1\right)$$

$$\sigma_r = -\frac{p r_1^2}{r_2^2 - r_1^2}\left(\frac{r_2^2}{R^2} - 1\right)$$

$$\sigma_x = -\frac{p r_1^2}{r_2^2 - r_1^2}$$